

Hedging Global Stock Markets with Bitcoin, Precious Metals, Copper, Crude Oil and Agricultural Commodities: Evidence from Multivariate Asymmetric GARCH Approach

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Abstract

Purpose: This paper examines effectiveness of hedging global stock market indexes with hedge asset futures of bitcoin, precious metals (gold, silver and palladium), copper, crude oil and agricultural products (wheat, orange juice and corn). Eleven global stock market indexes were selected from developed and developing economies: ASX 200, MSCI US, MSCI Europe, MSCI Japan, HSI, IBOVESPA, BSESN, SSECI, STI, TAIEX and TSXCI for assessing performance of hedging with the aforementioned asset futures.

Study design/methodology/approach: All weekly data are denominated in USD, collected from Eikon. The full sample period of the data runs from May 1, 2018, to March 2, 2023, with a total of 266 observations. We adopt asymmetric diagonal BEKK-GARCH model and the OLS model for analysis. The GARCH model can estimate the time-varying hedge ratios in order to improve hedging effectiveness. OLS model provides constant hedge ratios for comparison.

Findings: Under the BEKK-GARCH model, evidence of volatility spillover is stronger than that of return spillover. Furthermore, there is stronger evidence of asymmetric leverage effect in stock indexes than in hedge asset futures. Moreover, copper futures perform the best as a hedge asset, on average. Additionally, the average performance of hedging with hedge asset futures, against ASX 200 and TSXCI, is the highest among the stock market indexes examined. Furthermore, the GARCH model provides the highest level of average hedging effectiveness compared with OLS.

Practical Implications: This study assesses hedging performance of different combinations of stock market indexes and hedge assets. The optimal choice of hedge assets in different stock markets is illustrated. The overall results deliver a comprehensive picture of hedging strategies for portfolio managers. Hence, our study is related to the practice of decision sciences, providing insights into hedging processes and optimal solutions for portfolio managers and hedgers.

Keywords: hedge ratio, hedging effectiveness, asymmetric, BEKK-GARCH

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1. Introduction

Hedging is a risk management strategy that helps reduce potential losses from fluctuations in prices of securities, currencies or commodities by trading in hedge assets that are negatively or uncorrelated to the original investment. Hedging is most effective when it minimizes the variance of the hedged portfolio. Moreover, hedging can complement investment strategies by increasing expected returns per unit of risk exposed. Investors can also pursue more aggressive strategies when their investment risk is controlled within a limited range.

Use of futures contracts is considered an effective way of hedging market risk (Lien & Tse, 2002; Arenas-Falótico, & Scudiero, 2023). Among futures contracts traded as hedge assets, crude oil futures are very popular in practice (Yu et al., 2023). In addition, studies have found that metal futures have some positive correlation with stock markets and are suitable for hedging. For example, copper futures are widely used for hedging against some investment risks (Chen, 2023). Therefore, effectiveness of crude oil futures and copper futures in hedging against stocks is worthy of further study. Bitcoin is a new product in financial markets and its hedging effectiveness too is worth examining (Halipii, 2020), while precious metals futures and agricultural commodity futures are traded on various exchanges around the world, and their hedging ability is also worth studying (Hanif et al., 2023).

This research examines hedging effectiveness of cryptocurrency (bitcoin), precious metals (gold, silver and palladium), industrial metals (copper), energy commodity (crude oil) and agricultural commodities (wheat, orange juice and corn) futures in global stock markets. Eleven stock market indexes were selected for this study to represent the global stock markets: ASX 200 Index in Australia, MSCI USA Index, MSCI Europe Index, MSCI Japan Index, Bombay Stock Exchange Sensitive Index (BSESEN) in India, Bovespa Index (IBOVESPA) in Brazil, Hang Seng Index (HSI) in Hong Kong, Shanghai Stock Exchange Composite Index (SSECI) in China, Strait Times Index (STI) in Singapore, Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) in Taiwan and TSX Composite Index (TXCX) in Canada.

The rest of this paper is organized as follows. Section 2 presents literature review; Section 3 describes data, followed by research methods introduced in Section 4. Section 5 documents the empirical results, and finally Section 6 describes the main conclusions.

2. Literature Review

A large mass of extant literature has focused upon estimation of hedged ratios of returns on stocks to hedge assets in order to assess the hedging effectiveness. The hedge assets vary across different asset classes such as cryptocurrencies precious and industrial metals as well as energy and agricultural commodities.

Bitcoin is widely recognized as the most actively traded cryptocurrency, dominating the cryptocurrency markets in terms of trading volume and market capitalization (Mikhaylov, 2020; Fang et al., 2022). It is the most popular asset compared with other cryptocurrencies (for example, Ethereum, Litecoin, Tether, Solana and so on). Bitcoin is usually chosen not only as an investment vehicle but also as a potential hedge against stock market fluctuations (Xu & Kinkyo, 2023). Negative or zero correlations with other asset classes also make bitcoin attractive for hedging purposes (Wong et al., 2018).

Precious metals, particularly gold and silver, have been favoured as hedge assets in investment portfolios. Gold is widely regarded as a safe-haven asset and investors often hold gold during times of economic uncertainty or financial crisis (Rizvi et al., 2022). Moreover, gold tends to

retain its value during inflationary periods. As fiat currencies lose purchasing power, gold often increases in value, making it an effective hedge against inflation (Valadkhani et al., 2022). Gold has shown a negative correlation with stock markets movements, providing diversification and serving as a hedging asset in investment portfolios (Ali et al., 2020). Besides gold, silver too has a low correlation with other asset classes so that investors can diversify their portfolios with silver for risk management (Chiang, 2022; Paul et al., 2023). Silver can also be a hedge against inflation risk (Adekoya et al., 2021). Furthermore, like gold, silver is considered a safe haven during periods of economic uncertainty and financial distress (Dibooglu et al., 2022). Furthermore, like gold and silver, palladium also provides hedge advantages against inflation, market fluctuations and financial crises (Salisu et al., 2019; Naeem et al. 2022; Mensi et al., 2023).

Copper is a key industrial metal used in many sectors such as construction, renewable energy and electronics. Its unique characteristics make it attractive for hedging and diversification purposes (Chen, 2023). Copper is an important raw material and therefore its price may move together with inflation. As inflation rises, demand for copper in manufacturing and other industrial applications can also increase, thereby supporting the copper price. Hence, copper can be an inflation hedge effectively (Adekoya et al. 2023).

Crude oil is a prominent commodity, and its prices affect the international economy and trade. Oil prices and exchange rates can influence each other (Beckmann et al. 2020). Hedging against oil price and exchange rate fluctuations can be beneficial to investments in currencies and oil (Olstad et al. 2021). Moreover, safe haven, hedging and diversification functions of crude oil are important for oil exporters as well as importers (Ji et al., 2020; Liu & Lee, 2022; Ming et al., 2023). Due to low or negative correlations, especially during crisis periods, oil performs well in these functions (Liu et al., 2020). Some argue that hedging stocks with crude oil is possible (Batten et al., 2020) when crude oil and the stock markets are relatively independent of each other in terms of returns. Because of this, crude oil is used to hedge the risk of changes in stock markets (Feroni et al., 2017; Xu et al., 2020; Shahzad et al., 2022), especially when the stock markets are extremely volatile (Boubaker & Larbi, 2022).

Agricultural commodities and their prices move very differently from prices of non-farm financial assets. Prices of agricultural commodities are usually determined by their own demand and supply in their respective physical markets. Under this circumstance, agricultural commodities can be used for hedging against inflation and to protect investors against a decrease in purchasing power of money during inflationary periods (Spencer et al., 2018). Further, agricultural commodities have specific risk factors compared with financial products that include weather, supply chain disruptions and changing demand. As a result, specific patterns of agricultural markets lead to low correlation with stock markets so that agricultural commodities can offer diversification benefits for investors and provide opportunities for hedging the risk of stock portfolios (Hernandez et al., 2021; Gunera, 2023).

The findings of hedging performance of different asset classes reported in the literature vary because different stock markets and different sample periods are considered. This study examines hedging in major stock markets using the asymmetric GARCH method during the more recent periods, with different kinds of hedge assets mentioned above rather than one or two kinds only (e.g. Batten, et al., 2021; Bunditsakulporn, 2022; Chen, 2023; Kangalli Uyar, et al., 2022; Okorie, 2020). Hence, our results indicate a more comprehensive picture of hedging performance of hedge assets in stock markets.

3. Methodology

To examine the effectiveness of hedging stocks with selected asset futures, the paper employs the diagonal BEKK-TGARCH model (Engle and Kroner, 1995) for analysis as well as the ordinary least square (OLS) model.¹

3.1 Bivariate VAR Model

To begin with, we define $R_{f,t}$ and $R_{s,t}$ as returns of hedge asset futures and stock indexes, respectively, from time $t-1$ to time t . Then, we use Vector Autoregressive Model of order 1, VAR (1), to model the mean equations of $R_{f,t}$ and $R_{s,t}$.

The mean equations of $R_{f,t}$ and $R_{s,t}$ are modelled in bivariate VAR (1), as follows:

$$R_{f,t} = \mu_{f0} + \mu_{f1}R_{f,t-1} + \mu_{f2}R_{s,t-1} + \varepsilon_{f,t} \quad (1)$$

$$R_{s,t} = \mu_{s0} + \mu_{s1}R_{f,t-1} + \mu_{s2}R_{s,t-1} + \varepsilon_{s,t} \quad (2)$$

Where $\begin{bmatrix} \varepsilon_{f,t} \\ \varepsilon_{s,t} \end{bmatrix} | \Omega_{t-1} \sim N(0, H_t)$. $\varepsilon_{f,t}$ and $\varepsilon_{s,t}$ represent the return of the selected hedge asset futures and stock indexes, at time t , while Ω_{t-1} represents the information available at time $t-1$. Besides, H_t represents the conditional variance matrix of the error terms at time t . Also, μ_{fi} and μ_{si} , $i = 0, 1, 2$, denote the coefficients of mean equations (1) and (2). μ_{f2} and μ_{s1} are used to measure the return spillovers from stocks to hedge asset futures, and from hedge asset futures to stocks, respectively.²

3.2 Bivariate Diagonal BEKK-TGARCH Model

The diagonal BEKK model ensures that the variance-covariance matrix is always positive definite, which addresses the problem with the diagonal VECM model. A general form of the bivariate BEKK-TGARCH (1,1,1) is

$$H_t = C'C + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + B'H_{t-1}B + D\varepsilon_{t-1}I_{t-1}D'\varepsilon'_{t-1}I_{t-1} \quad (3)$$

where A , B and D are 2×2 diagonal matrices of parameters and C is an upper triangular matrix of parameters. H_t is a 2×2 conditional variance-covariance matrix, $I_{t-1} = 1$ if $\varepsilon_{t-1} < 0$ and = 0 otherwise. I_{t-1} is a dummy and is used to measure the asymmetric leverage effect.

Alternatively, Equation (3) can be shown as:

$$\begin{bmatrix} h_{11,t} & h_{12,t} \\ h_{12,t} & h_{22,t} \end{bmatrix} = CC' + \begin{bmatrix} a_{11} & 0 \\ 0 & a_{12} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1}^2 & \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{2,t-1}\varepsilon_{1,t-1} & \varepsilon_{2,t-1}^2 \end{bmatrix} \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix} + \begin{bmatrix} b_{11} & 0 \\ 0 & b_{12} \end{bmatrix} \\ \begin{bmatrix} h_{11,t-1} & h_{12,t-1} \\ h_{21,t-1} & h_{22,t-1} \end{bmatrix} \begin{bmatrix} b_{11} & 0 \\ 0 & b_{12} \end{bmatrix} + \begin{bmatrix} d_{11} & 0 \\ 0 & d_{12} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1}^2 & \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{2,t-1}\varepsilon_{1,t-1} & \varepsilon_{2,t-1}^2 \end{bmatrix} \begin{bmatrix} d_{11} & 0 \\ 0 & d_{12} \end{bmatrix} * I_{t-1}$$

where $C = \begin{bmatrix} C_{11} & C_{12} \\ 0 & C_{22} \end{bmatrix}$ is the matrix of intercept coefficients C_{11} , C_{12} and C_{22} , and a_{ij} denotes the coefficient of the previous volatility or the ARCH term $\varepsilon_{i,t-1}^2$ while b_{ij} is the coefficient of previous conditional variance or the GARCH term $h_{ij,t-1}$. In addition, d_{ij} is the coefficient to measure the asymmetric effects of negative and positive news. The bivariate diagonal BEKK-TGARCH (1,1) model can be then further represented as:

¹ DCC proposed by Engle (2002) is another popular kind of GARCH model. However, Caporin and McAleer (2013) discuss caveats and limits about the use of DCC and consider that BEKK model is more general as it allows for direct spillovers and feedback effects across conditional variance and covariances, as well as indirect spillovers and feedback effects across conditional correlations.

² Recent methods to measure time-varying spillovers include Coronado et al. (2020) and Feng et al. (2023).

$$h_{f,t} = C_{11} + a_{11}^2 \varepsilon_{1,t-1}^2 + b_{11}^2 h_{11,t-1} + d_{11}^2 \varepsilon_{1,t-1}^2 I_1 \quad (4)$$

$$h_{s,t} = C_{22} + a_{22}^2 \varepsilon_{2,t-1}^2 + b_{22}^2 h_{22,t-1} + d_{22}^2 \varepsilon_{2,t-1}^2 I_2 \quad (5)$$

$$h_{f,s,t} = C_{12} + a_{11} a_{22} \varepsilon_{1,t-1} \varepsilon_{2,t-1} + b_{11} b_{22} h_{12,t-1} + d_{11} d_{22} \varepsilon_{1,t-1} I_{1,t-1} \varepsilon_{2,t-1} I_{2,t-1} \quad (6)$$

where a_{11} and a_{22} are used to measure past information shocks on the current conditional variance-covariance of hedge asset futures returns and on stock returns, respectively. b_{11} and b_{22} measure the past conditional variance-covariance of hedge asset futures and stock returns on their own current conditional variance-covariances. d_{11} and d_{22} indicate the existence of the asymmetric leverage effect in the conditional variance-covariances.

3.3 Optimal Hedge Ratio

To investigate hedging effectiveness of hedge asset futures on global stock indexes, the optimal hedge ratio should be estimated to ensure that the variance of the hedged portfolio's returns is minimized. The return on hedged portfolio can be constructed as:

$$R_{p,t} = R_{s,t} - \beta_t R_{f,t} \quad (7)$$

where $R_{p,t}$, $R_{s,t}$ and $R_{f,t}$ are returns of hedged portfolios, global stock indexes and hedge asset futures, respectively, between time $t-1$ and time t . β_t represents the optimal hedge ratio between time $t-1$ and time t . The optimal hedge ratio is obtained from minimization of the conditional variance of return on hedged portfolio ($h_{p,t}$) (Baillie and Myers, 1991):

$$\beta_t = \frac{h_{f,s,t}}{h_{f,t}} \quad (8)$$

where $h_{f,s,t}$ and $h_{f,t}$ are time-varying conditional covariance between futures return and global stock index return, and conditional variance of futures return, respectively, at time t , which are estimated from Equations (4) and (6) using diagonal BEKK-TGARCH approach.

Furthermore, we use OLS to calculate static optimal hedge ratios for comparison:

$$R_{s,t} = \alpha + \beta R_{f,t} + \mu_t \quad (9)$$

where the OLS estimated hedge ratio is $\beta = \frac{h_{f,s}}{h_f}$. (10)

3.4 Hedging Effectiveness

Hedging effectiveness is a measure of the capability of a hedge asset to mitigate the risk level of a hedged portfolio (Ku et al. 2007). Hedging effectiveness can be written as:

$$\text{Hedging effectiveness} = \frac{\text{Variance}_{\text{unhedged}} - \text{Variance}_{\text{hedged}}}{\text{Variance}_{\text{unhedged}}} \quad (11)$$

where $\text{Variance}_{\text{unhedged}}$ denotes the variance of return on the unhedged portfolio while $\text{Variance}_{\text{hedged}}$ denotes the variance of return on the hedged portfolio.

4. Data

The dataset comprises global stock indexes and futures prices of nine hedge assets which include bitcoin (cryptocurrency), gold, silver and palladium (precious metals), copper (industrial metal), crude oil (energy commodity) as well as corn, orange juice and wheat (agricultural commodities). Further, we choose eleven representative stock indexes covering developed as well as developing economies: ASX 200, MSCI USA, MSCI Europe, MSCI Japan, IBOVESPA, BSESN, HSI, SSECI, STI, TAIEX and TSXCI.

All weekly data are collected from Eikon, denominated in USD. The full sample period of the data runs from May 1, 2018, to March 2, 2023, with a total of 266 observations. The data are taken into natural logarithms for analysis. Descriptive statistics for the return series are presented in Tables 1 and 2. Figures A1 and A2 show the price trends of all stock indexes and futures prices of hedge assets.

Table 1 Descriptive Statistics of Stock Index Return Series

	ASX200	MSCI US	MSCI Europe	MSCI Japan	IBOV-ESPA	BSESN	HSI	SSECI	STI	TAIEX	TSXCI
Mean	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Median	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00
Maximum	0.12	0.12	0.09	0.16	0.16	0.13	0.08	0.08	0.11	0.10	0.11
Minimum	-0.20	-0.16	-0.23	-0.17	-0.25	-0.15	-0.10	-0.10	-0.14	-0.12	-0.19
Std. Dev.	0.03	0.03	0.03	0.03	0.05	0.03	0.03	0.03	0.03	0.03	0.03
Skewness	-1.73	-0.83	-1.73	-0.20	-0.50	-0.45	-0.22	-0.37	-0.51	-0.68	-1.94
Kurtosis	14.05	8.66	15.60	12.86	5.88	7.31	3.33	3.67	9.21	5.35	15.22
Jarque-Bera	1481.23*	383.67*	1886.48*	1075.79*	102.30*	213.74*	3.34	10.87*	436.96*	81.16*	1813.85*
Obs.	266.00	266.00	266.00	266.00	266.00	266.00	266.00	266.00	266.00	266.00	266.00

Notes:

The Jarque-Bera statistic is used to test the null hypothesis that the data are from a normal distribution, and it is asymptotically chi-squared distribution with two degrees of freedom

*, **, *** denote statistical significance at the 1%, 5% and 10% level, respectively.

Table 2 Descriptive Statistics of Hedge Asset Futures Return Series

	BITCOIN	GOLD	SILVER	PALLADIUM	COPPER	CRUDE OIL	CORN	ORANGE JUICE	WHEAT
Mean	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Median	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Maximum	0.30	0.09	0.16	0.35	0.10	0.15	0.14	0.16	0.47
Minimum	0.54	-0.10	-0.17	-0.47	-0.12	-0.18	-0.25	-0.15	-0.21
Std. Dev.	0.10	0.02	0.04	0.06	0.03	0.00	0.04	0.04	0.05
Skewness	0.80	-0.15	-0.36	-1.00	-0.19	-0.39	-0.85	0.11	2.50
Kurtosis	6.39	6.45	6.64	18.43	4.28	6.88	9.60	4.04	28.71
Jarque-Bera	154.93*	132.61*	152.24*	2674.08*	19.67*	173.19*	512.51*	12.40*	7573.69*
Obs.	266.00	266.00	266.00	266.00	266.00	266.00	266.00	266.00	266.00

See notes to Table 1

Return on IBOVESPA (Table 1) exhibits the highest standard deviation, indicating that Brazil stock market is the most fluctuating compared with other sampled markets. Likewise, as shown in Table 2, return on bitcoin fluctuates the most among our sampled hedge assets. Also, all return series are not normally distributed.

4. Empirical Results

4.1 Unit root test

The unit root test is conducted by using the Augmented Dickey-Fuller (ADF) test (Table 4.1). Its results show that the price series are all non-stationary in level and their return series are stationary.

Table 3 Results of Unit Root Tests

Stock indexes	Level	1st Difference
ASX 200	-2.98	-10.00*
MSCI US	-2.36	-9.01*
MSCI Europe	-2.53	-9.00*
MSCI Japan	-2.13	-9.12*
IBOVESPA	-2.85	-10.06*
BSESN	-2.19	-6.35*
his	-2.52	-7.77*
SSECI	-2.33	-5.94*
STI	-2.04	-10.94*
TAIEX	-1.62	-3.14***
TSXCI	-2.64	-10.47*
Hedge Asset Futures		
Bitcoin	-2.16	-3.85**
Gold	-1.66	-5.43*
Silver	-2.27	-5.59*
Palladium	-0.48	-17.08*
Copper	-2.06	-3.60**
Crude oil	-1.52	-7.01*
Corn	-2.18	-4.66*
Orange Juice	-0.90	-16.17*
Wheat	-2.85	-9.49*

Notes:

An intercept and a linear trend are included in the ADF regression.

The critical value is -3.99 at the 1% level, -3.43 at the 5% level, -3.14 at the 10% level.

*, **, *** denote statistical significance at the 1%, 5%, 10% level, respectively

4.2 Estimation Results of Bivariate VAR-GARCH Model

4.2.1 Results of return spillovers in mean equations

Tables 4 to 12 present the results of return spillovers between stock and hedge asset futures markets from coefficients μ_{f2} and μ_{s1} in Equations (1) and (2). If μ_{f2} is positive (negative) and statistically significant, it means that an increase in lagged stock returns leads to a rise (fall) in current hedge asset returns in futures markets. It is found that the evidence of spillovers from the lagged stock returns to returns on hedge asset futures is scant, except the cases of hedging with gold (copper and crude oil), where the significant parameters μ_{f2} are negative (positive).

On the other hand, if μ_{s1} is positive (negative) and statistically significant, lagged asset returns in futures markets have positive (negative) impacts on current stock returns. Evidence of spillovers from the lagged asset futures returns to current stock returns is also scant. Then, current stock returns are not likely to be affected by the lagged returns in futures markets. In other words, evidence of spillovers between returns on stock indexes and hedge asset futures are weak.

Table 4 Estimation Results of Mean Equations: Hedging with Bitcoin Futures

		μ_{f0}	μ_{f1}	μ_{f2}	μ_{s0}	μ_{s1}	μ_{s2}
ASX 200	Coefficient	0.80	0.05	-0.21	0.21	0.03**	-0.07
	p-value	0.12	0.32	0.29	0.12	0.02	0.27
MSCI US	Coefficient	0.91	0.04	-0.05	0.52*	0.01	-0.09
	p-value	0.09	0.51	0.79	0.00	0.24	0.19
MSCI Europe	Coefficient	0.53	0.05	-0.03	0.14	0.02	-0.05
	p-value	0.34	0.43	0.90	0.29	0.16	0.46
MSCI Japan	Coefficient	0.70	0.03	-0.15	0.02	0.02	-0.06
	p-value	0.18	0.56	0.42	0.85	0.11	0.40
IBOVESPA	Coefficient	0.40	0.09	-0.08	0.01	0.00	0.03
	p-value	0.52	0.17	0.53	0.97	0.95	0.68
BSESN	Coefficient	0.54	0.05	-0.02	0.11	0.02	0.01
	p-value	0.36	0.39	0.92	0.44	0.16	0.82
HSI	Coefficient	0.35	0.07	-0.24	-0.14	0.02	-0.05
	p-value	0.56	0.27	0.24	0.43	0.26	0.37
SSECI	Coefficient	0.39	0.08	-0.49**	0.01	0.02	-0.01
	p-value	0.50	0.18	0.03	0.96	0.16	0.90
STI	Coefficient	0.38	0.03	0.02	-0.05	0.02***	0.10***
	p-value	0.49	0.59	0.94	0.64	0.09	0.08
TAIEX	Coefficient	0.51	0.07	-0.27	0.33**	0.01	0.07
	p-value	0.34	0.24	0.22	0.03	0.32	0.32
TSXCI	Coefficient	0.59	0.07	-0.13	0.21***	0.02	0.01
	p-value	0.28	0.24	0.59	0.08	0.17	0.89

Note:

The coefficients μ_{f2} and μ_{s1} measure the return spillovers from lagged stock returns to current asset futures returns, and from lagged asset futures returns to current stock returns, respectively.

*, **, *** indicate the 1%, 5% and 10% significance levels, respectively

Table 5 Estimation Results of Mean Equations: Hedging with Gold Futures

		μ_{f0}	μ_{f1}	μ_{f2}	μ_{s0}	μ_{s1}	μ_{s2}
ASX 200	Coefficient	0.16	-0.03	-0.08**	0.12	-0.02	0.03
	p-value	0.13	0.63	0.04	0.33	0.82	0.70
MSCI US	Coefficient	0.13	-0.06	-0.06***	0.39*	-0.02	0.00
	p-value	0.22	0.34	0.09	0.00	0.73	0.97
MSCI Europe	Coefficient	0.16	-0.05	-0.03	0.18	-0.03	-0.03
	p-value	0.14	0.47	0.48	0.16	0.73	0.61
MSCI Japan	Coefficient	0.13	-0.03	-0.10**	-0.04	-0.03	0.00
	p-value	0.22	0.61	0.02	0.77	0.66	0.97
HSI	Coefficient	0.09	-0.06	-0.08*	-0.10	0.07	-0.05
	p-value	0.42	0.42	0.01	0.56	0.50	0.49
IBOVESPA	Coefficient	0.10	-0.07	0.03	-0.04	-0.09	0.02
	p-value	0.39	0.36	0.23	0.89	0.57	0.75
BSESN	Coefficient	0.15	-0.05	-0.03	0.19	-0.10	0.04
	p-value	0.19	0.46	0.52	0.20	0.23	0.59
SSECI	Coefficient	0.14	-0.06	-0.07**	-0.02	-0.12	-0.03
	p-value	0.18	0.42	0.05	0.91	0.12	0.68
STI	Coefficient	0.15	-0.02	-0.12*	0.01	0.00	0.13
	p-value	0.17	0.82	0.01	0.95	0.98	0.04
TAIEX	Coefficient	0.15	-0.05	-0.04	0.29***	0.10	0.10
	p-value	0.14	0.45	0.30	0.06	0.17	0.11

TSXCI	Coefficient	0.16	-0.03	-0.04	0.14	-0.10	0.07
	p-value	0.15	0.63	0.34	0.26	0.15	0.25

See Note to Table 4.

Table 6 Estimation Results of Mean Equations: Hedging with Silver Futures

		μ_{f0}	μ_{f1}	μ_{f2}	μ_{s0}	μ_{s1}	μ_{s2}
ASX 200	Coefficient	0.07	-0.07	-0.06	0.11	-0.01	0.02
	p-value	0.70	0.36	0.43	0.40	0.90	0.79
MSCI US	Coefficient	0.06	-0.08	-0.07	0.35*	-0.02	-0.03
	p-value	0.73	0.21	0.35	0.00	0.55	0.66
MSCI Europe	Coefficient	0.08	-0.06	0.01	0.17	-0.03	-0.04
	p-value	0.65	0.35	0.93	0.18	0.54	0.50
MSCI Japan	Coefficient	0.07	-0.06	-0.12	-0.05	-0.01	-0.02
	p-value	0.72	0.40	0.15	0.69	0.77	0.79
HSI	Coefficient	-0.04	-0.05	-0.11***	-0.06	0.02	-0.04
	p-value	0.83	0.51	0.10	0.72	0.76	0.56
IBOVESPA	Coefficient	-0.07	-0.08	0.04	-0.05	-0.05	0.04
	p-value	0.72	0.27	0.39	0.84	0.49	0.56
BSESN	Coefficient	0.03	-0.07	0.01	0.18	-0.08**	0.05
	p-value	0.88	0.31	0.93	0.21	0.05	0.49
SSECI	Coefficient	0.02	-0.04	-0.05	0.11	0.03	0.00
	p-value	0.91	0.56	0.49	0.48	0.46	0.95
STI	Coefficient	0.05	-0.02	-0.13	0.01	0.00	0.12**
	p-value	0.80	0.73	0.16	0.96	0.93	0.04
TAIEX	Coefficient	0.11	-0.08	0.03	0.12	0.01	0.09
	p-value	0.59	0.22	0.70	0.46	0.83	0.18
TSXCI	Coefficient	0.07	-0.08	0.01	0.15	-0.08	0.08
	p-value	0.71	0.28	0.95	0.19	0.03	0.20

See Note to Table 4.

Table 7 Estimation Results of Mean Equations: Hedging with Palladium Futures

		μ_{f0}	μ_{f1}	μ_{f2}	μ_{s0}	μ_{s1}	μ_{s2}
ASX 200	Coefficient	0.54***	-0.07	-0.11	0.18	-0.01	0.02
	p-value	0.06	0.28	0.37	0.16	0.76	0.80
MSCI US	Coefficient	0.66**	-0.07	-0.12	0.43*	-0.01	-0.05
	p-value	0.02	0.23	0.27	0.00	0.51	0.50***
MSCI Europe	Coefficient	0.53**	-0.05	-0.07	0.21***	-0.02	-0.01
	p-value	0.05	0.49	0.53	0.09	0.46	0.86
MSCI Japan	Coefficient	0.40	-0.03	-0.21	-0.02	0.00	0.01
	p-value	0.16	0.66	0.11	0.89	0.98	0.94
HSI	Coefficient	0.58**	-0.07	-0.10	-0.02	-0.02	-0.01
	p-value	0.05	0.36	0.32	0.91	0.47	0.92
IBOVESPA	Coefficient	0.48	-0.08	0.02	0.04	-0.08***	0.05
	p-value	0.11	0.25	0.79	0.88	0.10	0.42
BSESN	Coefficient	0.55**	-0.04	-0.16	0.24	-0.05***	0.05
	p-value	0.05	0.56	0.20	0.09	0.07	0.45
STI	Coefficient	0.66**	-0.08	0.00	0.03	0.00	0.13**
	p-value	0.02	0.25	0.98	0.80	0.87	0.04
SSECI	Coefficient	0.56**	-0.07	-0.16	0.12	-0.01	-0.01
	p-value	0.05	0.29	0.15	0.45	0.75	0.93

TAIEX	Coefficient	0.63**	-0.05	-0.13	0.36*	-0.01	0.10
	p-value	0.02	0.38	0.24	0.01	0.79	0.15
TSXCI	Coefficient	0.54**	-0.03	-0.15	0.23**	-0.02	0.03
	p-value	0.05	0.61	0.22	0.05	0.40	0.61

See Note to Table 4.

Table 8 Estimation Results of Mean Equations: Hedging with Copper Futures

		μ_{f0}	μ_{f1}	μ_{f2}	μ_{s0}	μ_{s1}	μ_{s2}
ASX 200	Coefficient	0.26	-0.19*	0.21*	0.15	-0.08	0.07
	p-value	0.14	0.01	0.00	0.26	0.16	0.30
MSCI US	Coefficient	0.17	-0.17*	0.20*	0.38*	-0.08**	-0.01
	p-value	0.35	0.01	0.00	0.00	0.05	0.86
MSCI Europe	Coefficient	0.26	-0.20*	0.27*	0.16	-0.08	0.04
	p-value	0.12	0.00	0.00	0.22	0.11	0.57
MSCI Japan	Coefficient	0.19	-0.12***	0.15**	-0.07	-0.01	0.01
	p-value	0.30	0.10	0.02	0.60	0.89	0.85
HSI	Coefficient	0.17	-0.07	-0.002	-0.09	0.00	-0.06
	p-value	0.35	0.42	0.97	0.60	0.97	0.36
IBOVESPA	Coefficient	0.11	-0.17**	0.15*	-0.08	-0.25*	0.10
	p-value	0.55	0.02	0.00	0.78	0.01	0.17
BSESN	Coefficient	0.18	-0.14**	0.19*	0.18	-0.05	0.06
	p-value	0.34	0.04	0.00	0.25	0.35	0.40
SSECI	Coefficient	0.17	-0.04	0.04	0.04	-0.01	-0.00
	p-value	0.36	0.54	0.52	0.81	0.86	0.97
STI	Coefficient	0.18	-0.15**	0.18**	-0.02	-0.05	0.19*
	p-value	0.35	0.03	0.02	0.90	0.25	0.00
TAIEX	Coefficient	0.18	-0.20*	0.24*	0.32**	-0.00	0.04
	p-value	0.31	0.00	0.00	0.03	0.99	0.52
TSXCI	Coefficient	0.21	-0.22*	0.29*	0.19	-0.10**	0.09
	p-value	0.22	0.00	0.00	0.12	0.04	0.23

See Note to Table 4.

Table 9. Estimation Results of Mean Equations: Hedging with Oil Futures

		μ_{f0}	μ_{f1}	μ_{f2}	μ_{s0}	μ_{s1}	μ_{s2}
ASX 200	Coefficient	0.16	-0.18*	0.12***	0.11	-0.06	0.08
	p-value	0.30	0.01	0.06	0.42	0.21	0.25
MSCI US	Coefficient	0.18	-0.17*	0.11	0.37*	-0.08	-0.01
	p-value	0.24	0.01	0.14	0.00	0.08	0.87
MSCI Europe	Coefficient	0.18	-0.18*	0.17*	0.13	-0.05	0.01
	p-value	0.22	0.00	0.01	0.31	0.28	0.88
MSCI Japan	Coefficient	0.28	-0.18*	0.03	0.02	-0.07***	-0.01
	p-value	0.12	0.00	0.65	0.85	0.06	0.84
HSI	Coefficient	0.14	-0.12***	-0.07	-0.15	0.02	-0.03
	p-value	0.38	0.10	0.18	0.40	0.67	0.68
IBOVESPA	Coefficient	0.16	-0.13**	0.01	0.02	-0.10	0.03
	p-value	0.32	0.05	0.67	0.94	0.27	0.65
BSESN	Coefficient	0.19	-0.15**	0.10***	0.16	-0.03	0.06
	p-value	0.23	0.02	0.06	0.25	0.56	0.36
SSECI	Coefficient	0.19	-0.12***	-0.02	0.08	0.02	-0.01
	p-value	0.25	0.10	0.78	0.60	0.63	0.85
STI	Coefficient	0.19	-0.16*	0.11	0.00	0.01	0.17*
	p-value	0.22	0.01	0.14	0.99	0.79	0.01

TAIEX	Coefficient	0.19	-0.19*	0.16*	0.35**	-0.07***	0.12***
	p-value	0.22	0.00	0.01	0.02	0.07	0.07
TSXCI	Coefficient	0.21	-0.19*	0.16*	0.17	-0.04	0.07
	p-value	0.19	0.01	0.03	0.17	0.41	0.33

See Note to Table 4.

Table 10 Estimation Results of Mean Equations: Hedging with Corn Futures

		μ_{f0}	μ_{f1}	μ_{f2}	μ_{s0}	μ_{s1}	μ_{s2}
ASX 200	Coefficient	0.26	-0.1	-0.02	0.11	-0.01	0.02
	p-value	0.17	0.15	0.79	0.42	0.84	0.73
MSCI US	Coefficient	0.27	-0.11	0.07	0.38*	-0.01	0.04
	p-value	0.15	0.11	0.28	0.00	0.62	0.45
MSCI Europe	Coefficient	0.31***	-0.1	0.01	0.16	-0.03	-0.01
	p-value	0.09	0.14	0.77	0.24	0.3	0.9
MSCI Japan	Coefficient	0.26	-0.12***	0.01	0.01	-0.07*	-0.01
	p-value	0.18	0.09	0.89	0.95	0.00	0.89
HSI	Coefficient	0.33***	-0.13***	0.01	-0.04	-0.1**	-0.02
	p-value	0.10	0.07	0.9	0.8	0.03	0.73
IBOVESPA	Coefficient	0.26	-0.11	0.01	0.04	-0.04	0.04
	p-value	0.19	0.13	0.76	0.9	0.59	0.56
BSESN	Coefficient	0.2	-0.12***	-0.07	0.15	-0.01	0.03
	p-value	0.28	0.09	0.35	0.3	0.86	0.63
SSECI	Coefficient	0.31	-0.13**	0.02	0.06	-0.06	0.00
	p-value	0.11	0.05	0.75	0.70	0.15	0.95
STI	Coefficient	0.33***	-0.11	0.10	0.00	-0.03	0.14**
	p-value	0.08	0.13	0.12	0.99	0.34	0.03
TAIEX	Coefficient	0.29	-0.11	0.04	0.26***	0.01	0.06
	p-value	0.12	0.12	0.60	0.10	0.85	0.35
TSXCI	Coefficient	0.28	-0.11	-0.05	0.16	-0.02	0.05
	p-value	0.15	0.14	0.39	0.2	0.46	0.44

See Note to Table 4.

Table 11 Estimation Results of Mean Equations: Hedging with Orange Juice Futures

		μ_{f0}	μ_{f1}	μ_{f2}	μ_{s0}	μ_{s1}	μ_{s2}
ASX 200	Coefficient	0.10	0.00	-0.11	0.11	0.01	0.03
	p-value	0.66	0.98	0.13	0.42	0.86	0.68
MSCI US	Coefficient	0.08	-0.01	-0.11	0.30**	0.02	-0.01
	p-value	0.75	0.91	0.13	0.02	0.49	0.92
MSCI Europe	Coefficient	0.12	0.00	0.04	0.07	0.00	-0.02
	p-value	0.63	0.98	0.54	0.59	0.90	0.81
MSCI Japan	Coefficient	0.16	0.01	-0.03	-0.07	0.00	0.01
	p-value	0.53	0.94	0.72	0.59	0.91	0.93
his	Coefficient	0.12	0.01	-0.08	-0.11	-0.06	-0.02
	p-value	0.63	0.91	0.35	0.54	0.18	0.80
IBOVESPA	Coefficient	0.12	0.01	-0.02	-0.09	-0.11***	0.04
	p-value	0.63	0.91	0.65	0.74	0.10	0.54
BSESN	Coefficient	0.04	0.00	0.10	0.11	0.02	0.03
	p-value	0.86	0.98	0.21	0.49	0.48	0.65
SSECI	Coefficient	0.21	0.00	-0.23*	-0.03	-0.01	0.01
	p-value	0.36	0.99	0.01	0.81	0.76	0.91

STI	Coefficient	0.10	0.01	-0.05	-0.02	-0.02	0.15**
	p-value	0.67	0.93	0.55	0.84	0.40	0.03
TAIEX	Coefficient	0.12	-0.01	-0.16	0.26	-0.04	0.09
	p-value	0.63	0.88	0.09	0.11	0.32	0.16
TSXCI	Coefficient	0.12	0.00	-0.11	0.10	-0.01	0.05
	p-value	0.62	0.96	0.12	0.44	0.80	0.42

See Note to Table 4.

Table 12 Estimation Results of Mean Equations: Hedging with Wheat Futures

		μ_{f0}	μ_{f1}	μ_{f2}	μ_{s0}	μ_{s1}	μ_{s2}
ASX 200	Coefficient	0.19	-0.11***	-0.18**	0.05	-0.01	0.07
	p-value	0.43	0.10	0.03	0.71	0.84	0.32
MSCI US	Coefficient	0.24	-0.13**	-0.17**	0.36	0.01	-0.01
	p-value	0.34	0.03	0.04	0.01	0.74	0.84
MSCI Europe	Coefficient	0.19	-0.08	-0.09	0.11	-0.03	0.00
	p-value	0.43	0.20	0.23	0.41	0.31	0.96
MSCI Japan	Coefficient	0.15	-0.09	-0.11	-0.02	-0.07	0.01
	p-value	0.53	0.11	0.23	0.85	0.01	0.92
HSI	Coefficient	0.16	-0.10	-0.08	-0.11	-0.11	-0.04
	p-value	0.50	0.12	0.34	0.52	0.01	0.53
IBOVESPA	Coefficient	0.12	-0.07	-0.05	-0.06	-0.04	0.02
	p-value	0.61	0.31	0.27	0.83	0.47	0.80
BSESN	Coefficient	0.08	-0.11***	-0.04	0.13	-0.01	0.04
	p-value	0.76	0.09	0.60	0.39	0.67	0.52
SSECI	Coefficient	0.12	-0.10	-0.05	0.02	-0.03	0.00
	p-value	0.63	0.13	0.58	0.92	0.29	0.99
STI	Coefficient	0.21	-0.09	-0.02	-0.01	-0.01	0.14**
	p-value	0.38	0.14	0.78	0.96	0.67	0.02
TAIEX	Coefficient	0.21	-0.09	-0.13	0.27	-0.02	0.08
	p-value	0.38	0.13	0.14	0.08	0.52	0.19
TSXCI	Coefficient	0.23	-0.09	-0.17	0.10	-0.01	0.07
	p-value	0.33	0.13	0.03	0.41	0.59	0.33

See Note to Table 4.

4.2.2 Results of volatility spillovers and asymmetry in variance-covariance equations

Tables 13 to 21 report the results of volatility spillovers and asymmetric leverage effect on the conditional variance-covariance in Equations (4)-(6). Most of the a_{11} coefficients are found to be statistically significant (except in cases of bitcoin and palladium) and almost all of a_{22} coefficients are insignificant. Hence, past information shocks likely have a significant spillover effect on the current conditional variance of hedge asset futures returns but not on stock returns. Most of the b_{11} and b_{22} coefficients are significant and therefore there is strong evidence to the effect that past conditional variance-covariance of returns on hedge asset futures and stocks have impacts on their own current conditional variance-covariances. Moreover, almost all d_{22} coefficients are significant (except in the case of bitcoin) but almost all d_{11} coefficients are insignificant. Then, strong evidence of asymmetric leverage effect exists in the conditional variance of stock markets under study,³ but such evidence of asymmetry in hedge asset futures markets is scant. Further, when a_{22} and d_{11} coefficients are mostly insignificant, the evidence

³ There is no evidence of asymmetry in the conditional variance of SSECI **except that** oil is a hedge asset.

for overall impacts of past shocks and asymmetric leverage on the conditional covariance are accordingly weak.

Table 13 Estimation Results of Variance-Covariance Equations: Hedging with Bitcoin Futures

		C_{11}	C_{12}	C_{22}	a_{11}	a_{22}
ASX 200	Coefficient	-1.53*	0.99***	0.81***	0.01	0.40**
	p-value	0.00	0.07	0.06	0.90	0.04
MSCI US	Coefficient	-1.68*	1.34**	0.71**	0.01	0.65*
	p-value	0.00	0.03	0.05	0.95	0.00
MSCI Europe	Coefficient	16.12	1.01	0.51**	0.26**	0.08
	p-value	0.32	0.15	0.03	0.02	0.73
MSCI Japan	Coefficient	-1.64**	1.73***	1.92***	0.01	0.47**
	p-value	0.04	0.09	0.06	0.92	0.03
HSI	Coefficient	18.88	0.98	0.54	-0.23*	-0.05
	p-value	0.40	0.30	0.35	0.05	0.82
IBOVESPA	Coefficient	61.46	11.30	3.54***	-0.17	0.18
	p-value	0.46	0.12	0.07	0.36	0.44
BSESN	Coefficient	64.22	7.01	0.93**	-0.09	0.02
	p-value	0.68	0.26	0.05	0.77	0.98
SSECI	Coefficient	18.93	1.71	4.26	0.21	0.22
	p-value	0.45	0.45	0.46	0.11	0.19
STI	Coefficient	16.63	1.59**	0.86*	-0.26**	0.04
	p-value	0.31	0.04	0.01	0.02	0.85
TAIEX	Coefficient	19.38	9.19*	2.88**	-0.24***	0.28
	p-value	0.42	0.00	0.02	0.08	0.31
TSXCI	Coefficient	15.67	1.35	0.51**	0.23**	0.17
	p-value	0.40	0.10	0.02	0.05	0.49
		d_{11}	d_{22}	b_{11}	b_{22}	
ASX 200	Coefficient	0.00	0.00	1.01*	0.85*	
	p-value	1.00	1.00	0.00	0.00	
MSCI US	Coefficient	0.00	0.03	1.01*	0.74*	
	p-value	1.00	1.00	0.00	0.00	
MSCI Europe	Coefficient	0.08	0.40	0.88*	0.91*	
	p-value	0.71	0.00	0.00	0.00	
MSCI Japan	Coefficient	0.00	0.00	1.01*	0.66*	
	p-value	1.00	1.00	0.00	0.00	
HSI	Coefficient	-0.01	0.32*	0.86*	0.94*	
	p-value	0.97	0.00	0.00	0.00	
IBOVESPA	Coefficient	-0.11	0.44*	-0.55	0.85*	
	p-value	0.73	0.01	0.51	0.00	
BSESN	Coefficient	-0.09	0.54*	-0.56	0.86*	
	p-value	0.67	0.00	0.70	0.00	
SSECI	Coefficient	-0.10	0.23	0.86*	0.61	
	p-value	0.68	0.41	0.00	0.33	
STI	Coefficient	0.05	0.62*	0.87*	0.82*	
	p-value	0.77	0.00	0.00	0.00	
TAIEX	Coefficient	-0.05	0.51	0.86*	-0.65*	
	p-value	0.85	0.01	0.00	0.00	
TSXCI	Coefficient	0.05	0.55*	0.89*	0.86*	
	p-value	0.74	0.00	0.00	0.00	

Note:

The coefficients a_{11} and a_{22} measure lagged shocks on the current conditional variance-covariance of asset futures returns and on stock returns, respectively. b_{11} and b_{22} coefficients measure the lagged conditional variance-covariance of asset futures and stock returns on their own current conditional variance-covariances. d_{11} and d_{22} coefficients indicate the existence of the asymmetric leverage effect in the conditional variance-covariances.

*, **, *** indicate the 1%, 5% and 10% significance levels, respectively

Table 14 Estimation Results of Variance-Covariance Equations: Hedging with Gold Futures

		C_{11}	C_{12}	C_{22}	a_{11}	a_{22}
ASX 200	Coefficient	0.12	0.08	0.50**	0.23*	0.08
	p-value	0.21	0.25	0.03	0.00	0.67
MSCI US	Coefficient	0.24***	0.25***	1.76*	0.30*	0.38*
	p-value	0.06	0.08	0.00	0.00	0.00
MSCI Europe	Coefficient	1.41***	0.33	0.69***	0.39*	0.20
	p-value	0.09	0.13	0.06	0.00	0.19
MSCI Japan	Coefficient	0.00	0.00	0.00	0.19**	-0.02
	p-value	0.94	0.94	0.94	0.00	0.83
HSI	Coefficient	0.18	0.16	1.88	0.28*	0.24
	p-value	0.14	0.33	0.24	0.00	0.18
IBOVESPA	Coefficient	0.15	0.37**	3.06***	-0.26*	0.16
	p-value	0.14	0.04	0.07	0.00	0.37
BSESN	Coefficient	0.20	0.09	0.83**	0.23*	-0.01
	p-value	0.17	0.31	0.04	0.00	0.96
SSECI	Coefficient	0.05	0.05	0.05	0.26*	0.06
	p-value	0.21	0.21	0.21	0.00	0.52
STI	Coefficient	0.19	0.18	0.88**	0.24*	0.08
	p-value	0.20	0.12	0.05	0.01	0.64
TAIEX	Coefficient	0.22	0.31	3.23**	0.21***	0.10
	p-value	0.18	0.23	0.05	0.07	0.76
TSXCI	Coefficient	0.15	0.09	0.45**	0.22*	0.06
	p-value	0.20	0.16	0.04	0.00	0.81
		d_{11}	d_{22}	b_{11}	b_{22}	
ASX 200	Coefficient	0.14	0.54*	0.95*	0.88*	
	p-value	0.20	0.00	0.00	0.00	
MSCI US	Coefficient	0.08	1.15*	0.92*	0.45*	
	p-value	0.63	0.00	0.00	0.00	
MSCI Europe	Coefficient	0.20	0.47*	0.70*	0.86*	
	p-value	0.41	0.00	0.00	0.00	
MSCI Japan	Coefficient	0.12	0.24*	0.98*	0.98*	
	p-value	0.30	0.00	0.00	0.00	
HSI	Coefficient	0.08	0.31***	0.93*	0.83*	
	p-value	0.72	0.06	0.00	0.00	
IBOVESPA	Coefficient	0.08	0.46*	0.94*	0.86*	
	p-value	0.60	0.00	0.00	0.00	
BSESN	Coefficient	0.08	0.54*	0.94*	0.86*	
	p-value	0.54	0.00	0.00	0.00	
SSECI	Coefficient	0.00	0.05	0.96*	0.99*	
	p-value	1.00	0.21	0.00	0.00	
STI	Coefficient	0.11	0.62*	0.95*	0.81*	
	p-value	0.33	0.00	0.00	0.00	

TAIEX	Coefficient	0.19	0.53*	0.94*	0.63*	
	p-value	0.25	0.00	0.00	0.00	
TSXCI	Coefficient	0.13	0.56*	0.95*	0.87*	
	p-value	0.29	0.00	0.00	0.00	

See Note to Table 13.

Table 15 Estimation Results of Variance-Covariance Equations: Hedging with Silver Futures

		C_{11}	C_{12}	C_{22}	a_{11}	a_{22}
ASX 200	Coefficient	0.34	0.18	0.58**	-0.19**	0.10
	p-value	0.11	0.17	0.02	0.04	0.52
MSCI US	Coefficient	0.27	0.08	0.41**	-0.23*	0.05
	p-value	0.15	0.47	0.03	0.01	0.77
MSCI Europe	Coefficient	0.42	0.17	0.45**	-0.27*	0.02
	p-value	0.13	0.18	0.05	0.00	0.89
MSCI Japan	Coefficient	0.37***	0.13	1.01*	-0.20**	0.14
	p-value	0.09	0.44	0.01	0.02	0.44
HSI	Coefficient	0.40***	0.40	2.58	0.32*	0.26
	p-value	0.06	0.31	0.20	0.00	0.14
IBOVESPA	Coefficient	11.00	17.72	17.72	0.32	0.52
	p-value	0.40	0.07	0.30	0.49	0.13
BSESN	Coefficient	0.36	0.10	0.73**	-0.28*	-0.03
	p-value	0.11	0.49	0.04	0.00	0.86
SSECI	Coefficient	0.27	0.62	4.41	0.28*	0.20
	p-value	0.14	0.50	0.36	0.00	0.38
STI	Coefficient	0.32	0.26	0.80**	-0.26*	0.03
	p-value	0.12	0.14	0.02	0.01	0.90
TAIEX	Coefficient	0.36	0.29	1.74	0.32*	0.29
	p-value	0.11	0.31	0.17	0.00	0.14
TSXCI	Coefficient	0.36	0.20***	0.41**	-0.24*	-0.01
	p-value	0.15	0.08	0.03	0.01	0.96
		d_{11}	d_{22}	b_{11}	b_{22}	
ASX 200	Coefficient	0.30*	0.52*	0.95*	0.88*	
	p-value	0.00	0.00	0.00	0.00	
MSCI US	Coefficient	0.25*	0.62*	0.95*	0.86*	
	p-value	0.01	0.00	0.00	0.00	
MSCI Europe	Coefficient	0.23***	0.45*	0.94*	0.91*	
	p-value	0.06	0.00	0.00	0.00	
MSCI Japan	Coefficient	0.28*	0.71*	0.95*	0.78*	
	p-value	0.00	0.00	0.00	0.00	
HSI	Coefficient	0.11	0.30	0.93*	0.78*	
	p-value	0.62	0.12	0.00	0.00	
IBOVESPA	Coefficient	0.01	0.01	0.94*	0.69*	
	p-value	1.00	1.00	0.00	0.01	
BSESN	Coefficient	0.16	0.54*	0.94*	0.87*	
	p-value	0.27	0.00	0.00	0.00	
SSECI	Coefficient	0.21	0.18	0.94*	0.61	
	p-value	0.19	0.62	0.00	0.26	
STI	Coefficient	0.24**	0.62*	0.94*	0.82*	
	p-value	0.04	0.00	0.00	0.00	
TAIEX	Coefficient	0.00	0.00	0.94*	0.82*	
	p-value	1.00	1.00	0.00	0.00	

TSXCI	Coefficient	0.25*	0.53*	0.94*	0.89*	
	p-value	0.01	0.00	0.00	0.00	

See Note to Table 13.

Table 16 Estimation Results of Variance-Covariance Equations: Hedging with Palladium Futures

		C ₁₁	C ₁₂	C ₂₂	a ₁₁	a ₂₂
ASX 200	Coefficient	24.71	0.54	6.43	0.43	0.59
	p-value	0.39	0.27	0.27	0.21	0.12
MSCI US	Coefficient	1.85***	0.24	0.24*	0.22	0.28**
	p-value	0.08	0.12	0.01	0.21	0.04
MSCI Europe	Coefficient	5.26*	5.26*	5.26*	0.24***	0.26*
	p-value	0.00	0.00	0.00	0.08	0.01
MSCI Japan	Coefficient	5.44	1.72	3.39*	0.13	0.41*
	p-value	0.25	0.07	0.01	0.25	0.01
HSI	Coefficient	2.94***	0.60	0.91	0.29*	0.01
	p-value	0.09	0.14	0.33	0.00	0.94
IBOVESPA	Coefficient	0.75***	0.75***	0.75***	-0.24*	0.23
	p-value	0.06	0.06	0.06	0.00	0.13
BSESN	Coefficient	3.93	0.89	0.89**	0.29*	0.14
	p-value	0.11	0.15	0.05	0.00	0.37
SSECI	Coefficient	2.13***	1.31	3.33	0.27*	-0.19
	p-value	0.07	0.45	0.56	0.01	0.25
STI	Coefficient	3.28	0.56	0.75**	0.27*	0.08
	p-value	0.14	0.14	0.03	0.00	0.67
TAIEX	Coefficient	0.01	0.01	0.01	-0.05	0.05
	p-value	0.92	0.92	0.92	0.67	0.95
TSXCI	Coefficient	24.42	5.72	0.41	0.53*	0.21
	p-value	0.12	0.14	0.04	0.01	0.30
		d ₁₁	d ₂₂	b ₁₁	b ₂₂	
ASX 200	Coefficient	0.00	0.00	0.83*	0.76*	
	p-value	1.00	1.00	0.00	0.00	
MSCI US	Coefficient	0.19**	0.66*	0.93*	0.86*	
	p-value	0.04	0.00	0.00	0.00	
MSCI Europe	Coefficient	0.59	0.59**	0.87*	-0.30*	
	p-value	0.21	0.02	0.00	0.01	
MSCI Japan	Coefficient	0.24**	0.76*	0.88*	0.40***	
	p-value	0.02	0.00	0.00	0.10	
HSI	Coefficient	0.09	0.34*	0.90*	0.92*	
	p-value	0.61	0.00	0.00	0.00	
IBOVESPA	Coefficient	0.13	0.39*	0.95*	0.93*	
	p-value	0.33	0.00	0.00	0.00	
BSESN	Coefficient	0.20	0.57*	0.87*	0.85*	
	p-value	0.11	0.00	0.00	0.00	
SSECI	Coefficient	0.11	0.15	0.92*	0.75	
	p-value	0.70	0.69	0.00	0.12	
STI	Coefficient	-0.12	-0.60*	0.90*	0.84*	
	p-value	0.42	0.00	0.00	0.00	
TAIEX	Coefficient	0.06	0.32*	1.00*	0.98*	
	p-value	0.66	0.01	0.00	0.00	
TSXCI	Coefficient	0.00	0.00	0.83*	0.78*	
	p-value	1.00	1.00	0.00	0.00	

See Note to Table 13.

Table 17. Estimation Results of Variance-Covariance Equations: Hedging with Copper Futures

		C_{11}	C_{12}	C_{22}	a_{11}	a_{22}
ASX 200	Coefficient	6.92*	4.13*	0.94*	0.43*	-0.10
	p-value	0.00	0.00	0.00	0.00	0.55
MSCI US	Coefficient	5.44*	1.16**	0.89*	0.30*	-0.28**
	p-value	0.01	0.02	0.01	0.01	0.07
MSCI Europe	Coefficient	6.75*	4.47*	0.56**	0.41**	-0.03
	p-value	0.00	0.00	0.02	0.02	0.90
MSCI Japan	Coefficient	6.18*	3.25*	1.35*	-0.37*	0.00
	p-value	0.03	0.00	0.00	0.00	0.98
HSI	Coefficient	7.48*	5.36**	1.80	0.40*	-0.03
	p-value	0.00	0.01	0.27	0.00	0.86
IBOVESPA	Coefficient	5.44*	2.15**	3.67**	0.24	0.00
	p-value	0.00	0.05	0.03	0.16	1.00
BSESN	Coefficient	5.52***	0.93	0.90**	0.25***	0.04
	p-value	0.06	0.14	0.03	0.09	0.89
SSECI	Coefficient	7.08*	3.26**	2.56	0.57*	0.15
	p-value	0.00	0.06	0.74	0.00	0.21
STI	Coefficient	-0.24*	0.46**	0.82*	0.02	-0.01
	p-value	0.00	0.03	0.01	0.94	0.99
TAIEX	Coefficient	-0.22*	0.88	2.28**	0.00	0.12
	p-value	0.00	0.14	0.08	0.99	0.81
TSXCI	Coefficient	5.23**	1.37**	0.64*	0.33**	-0.01
	p-value	0.01	0.03	0.00	0.02	0.96
		d_{11}	d_{22}	b_{11}	b_{22}	
ASX 200	Coefficient	0.44**	0.52*	-0.26	0.85*	
	p-value	0.04	0.00	0.34	0.00	
MSCI US	Coefficient	0.48*	0.68*	0.51**	0.75*	
	p-value	0.01	0.00	0.03	0.00	
MSCI Europe	Coefficient	0.37	0.39*	-0.36	0.91*	
	p-value	0.20	0.00	0.24	0.00	
MSCI Japan	Coefficient	0.16	0.71*	-0.47	0.74*	
	p-value	0.54	0.00	0.16	0.00	
HSI	Coefficient	0.15	0.34*	-0.21	0.86*	
	p-value	0.66	0.01	0.63	0.00	
IBOVESPA	Coefficient	0.49*	0.51*	0.51**	0.84*	
	p-value	0.01	0.00	0.03	0.00	
BSESN	Coefficient	0.33	0.53*	0.55**	0.86*	
	p-value	0.12	0.00	0.06	0.00	
SSECI	Coefficient	0.00	0.00	0.17	0.80	
	p-value	1.00	1.00	0.73	0.23	
STI	Coefficient	0.04	0.54*	1.01*	0.85*	
	p-value	0.71	0.00	0.00	0.00	
TAIEX	Coefficient	0.05	0.46*	1.01*	0.79*	
	p-value	0.68	0.01	0.00	0.00	
TSXCI	Coefficient	0.43**	0.53*	0.52**	0.85*	
	p-value	0.04	0.00	0.03	0.00	

See Note to Table 13.

Table 18 Estimation Results of Variance-Covariance Equations: Hedging with Oil Futures

		C ₁₁	C ₁₂	C ₂₂	a ₁₁	a ₂₂
ASX 200	Coefficient	0.50	0.21	0.57**	0.34*	0.10
	p-value	0.11	0.12	0.02	0.00	0.42
MSCI US	Coefficient	0.53	0.18	0.47**	0.39*	0.09
	p-value	0.11	0.17	0.02	0.00	0.54
MSCI Europe	Coefficient	0.48	0.25***	0.52**	0.37*	0.08
	p-value	0.15	0.09	0.03	0.00	0.52
MSCI Japan	Coefficient	-0.38*	0.89***	2.57***	0.09	0.09
	p-value	0.00	0.10	0.06	0.17	0.84
HSI	Coefficient	0.57	0.21	1.14	0.47*	0.30**
	p-value	0.12	0.32	0.30	0.00	0.05
IBOVESPA	Coefficient	0.74***	0.60***	2.72**	0.32*	0.02
	p-value	0.08	0.07	0.05	0.01	0.91
BSESN	Coefficient	0.42	0.08	0.61**	0.37*	0.08
	p-value	0.13	0.49	0.05	0.00	0.51
SSECI	Coefficient	0.08	0.08	0.08	0.35*	0.17
	p-value	0.23	0.23	0.23	0.00	0.85
STI	Coefficient	0.42	0.24***	0.57*	0.35*	0.11
	p-value	0.13	0.08	0.01	0.00	0.50
TAIEX	Coefficient	0.41	0.23	1.47	0.33*	0.03
	p-value	0.17	0.28	0.19	0.00	0.89
TSXCI	Coefficient	0.70***	0.44**	0.62*	0.36*	0.05
	p-value	0.08	0.02	0.01	0.00	0.74
		d ₁₁	d ₂₂	b ₁₁	b ₂₂	
ASX 200	Coefficient	0.30*	0.48*	0.90*	0.89*	
	p-value	0.01	0.00	0.00	0.00	
MSCI US	Coefficient	0.31**	0.57*	0.88*	0.86*	
	p-value	0.02	0.00	0.00	0.00	
MSCI Europe	Coefficient	0.21	0.42*	0.90*	0.90*	
	p-value	0.25	0.00	0.00	0.00	
MSCI Japan	Coefficient	0.07	0.68*	1.01*	0.72*	
	p-value	0.49	0.00	0.00	0.00	
HSI	Coefficient	0.00	0.26	0.88*	0.88*	
	p-value	1.00	0.16	0.00	0.00	
IBOVESPA	Coefficient	0.33	0.49*	0.88*	0.88*	
	p-value	0.01	0.00	0.00	0.00	
BSESN	Coefficient	0.25**	0.48*	0.90*	0.89*	
	p-value	0.03	0.00	0.00	0.00	
SSECI	Coefficient	0.14**	0.14***	0.94*	0.99*	
	p-value	0.33	0.08	0.00	0.00	
STI	Coefficient	0.19	0.57*	0.92*	0.86*	
	p-value	0.25	0.00	0.00	0.00	
TAIEX	Coefficient	0.17	0.35*	0.92*	0.85*	
	p-value	0.33	0.01	0.00	0.00	
TSXCI	Coefficient	0.28**	0.52*	0.88*	0.87*	
	p-value	0.03	0.00	0.00	0.00	

See Note to Table 13.

Table 19. Estimation Results of Variance-Covariance Equations: Hedging with Corn Futures

		C ₁₁	C ₁₂	C ₂₂	a ₁₁	a ₂₂
ASX 200	Coefficient	2.65**	0.07	0.57**	0.60*	0.12
	p-value	0.04	0.83	0.02	0.00	0.47
MSCI US	Coefficient	2.68**	0.07	0.48**	0.58*	0.23
	p-value	0.04	0.76	0.03	0.00	0.20
MSCI Europe	Coefficient	2.43***	0.05	0.52**	0.62*	0.14
	p-value	0.06	0.85	0.04	0.00	0.37
MSCI Japan	Coefficient	2.51**	0.17	1.09*	0.60*	0.18
	p-value	0.04	0.60	0.01	0.00	0.27
HSI	Coefficient	2.42**	0.27	1.01	0.54*	0.18
	p-value	0.04	0.35	0.31	0.00	0.16
IBOVESPA	Coefficient	2.26**	0.84	3.64**	0.52*	-0.04
	p-value	0.05	0.12	0.04	0.00	0.81
BSESN	Coefficient	2.20**	-0.24	0.84***	0.56*	0.08
	p-value	0.04	0.29	0.07	0.00	0.53
SSECI	Coefficient	2.56**	0.25	5.28	0.57*	0.16
	p-value	0.05	0.64	0.50	0.00	0.30
STI	Coefficient	2.32***	0.08	0.79*	0.57*	0.09
	p-value	0.06	0.73	0.01	0.00	0.62
TAIEX	Coefficient	2.59**	0.05	1.63***	0.59*	0.13
	p-value	0.05	0.88	0.09	0.00	0.53
TSXCI	Coefficient	2.79**	-0.06	0.51**	0.57*	0.00
	p-value	0.04	0.83	0.02	0.00	1.00
		d ₁₁	d ₂₂	b ₁₁	b ₂₂	
ASX 200	Coefficient	0.04	0.61*	0.72*	0.86*	
	p-value	0.93	0.00	0.00	0.00	
MSCI US	Coefficient	0.07	-0.70*	0.73*	0.82*	
	p-value	0.80	0.00	0.00	0.00	
MSCI Europe	Coefficient	-0.09	0.48*	0.73*	0.89*	
	p-value	0.84	0.00	0.00	0.00	
MSCI Japan	Coefficient	-0.04	0.74*	0.73*	0.76*	
	p-value	0.91	0.00	0.00	0.00	
HSI	Coefficient	-0.20	0.36*	0.74*	0.90*	
	p-value	0.47	0.00	0.00	0.00	
IBOVESPA	Coefficient	0.25	0.57*	0.75*	0.84*	
	p-value	0.33	0.00	0.00	0.00	
BSESN	Coefficient	0.34	0.56*	0.74*	0.86*	
	p-value	0.14	0.00	0.00	0.00	
SSECI	Coefficient	-0.17	0.13	0.73*	0.55	
	p-value	0.73	0.73	0.00	0.56	
STI	Coefficient	0.10	0.62*	0.75*	0.83*	
	p-value	0.74	0.00	0.00	0.00	
TAIEX	Coefficient	-0.10	0.47*	0.74*	0.81*	
	p-value	0.81	0.00	0.00	0.00	
TSXCI	Coefficient	0.28	0.65*	0.72*	0.85*	
	p-value	0.34	0.00	0.00	0.00	

See Note to Table 13.

Table 20. Estimation Results of Variance-Covariance Equations: Hedging with Orange Juice Futures

		C_{11}	C_{12}	C_{22}	a_{11}	a_{22}
ASX 200	Coefficient	0.80	0.29	0.66**	-0.33*	0.33***
	p-value	0.32	0.22	0.02	0.00	0.07
MSCI US	Coefficient	0.83	0.08	1.03**	-0.25*	0.36**
	p-value	0.29	0.74	0.02	0.01	0.04
MSCI Europe	Coefficient	1.09	0.15	0.55**	0.30*	-0.10
	p-value	0.29	0.43	0.02	0.00	0.65
MSCI Japan	Coefficient	0.89	0.09	1.19*	0.27*	-0.05
	p-value	0.30	0.74	0.00	0.01	0.85
HSI	Coefficient	0.91	0.19	1.26	0.27*	-0.18
	p-value	0.28	0.41	0.32	0.00	0.27
IBOVESPA	Coefficient	0.94	0.44	3.25**	0.27*	0.06
	p-value	0.26	0.23	0.02	0.00	0.76
BSESN	Coefficient	0.79	0.22	0.92**	0.28*	-0.12
	p-value	0.24	0.26	0.03	0.00	0.53
SSECI	Coefficient	0.50	0.10	1.49	0.24*	0.23***
	p-value	0.41	0.62	0.48	0.02	0.08
STI	Coefficient	0.97	-0.01	0.69*	0.27*	0.02
	p-value	0.29	0.94	0.00	0.01	0.93
TAIEX	Coefficient	1.11	1.35	1.82***	0.28**	0.10
	p-value	0.34	0.31	0.07	0.02	0.70
TSXCI	Coefficient	0.86	0.14	0.54**	-0.27*	0.19
	p-value	0.32	0.40	0.02	0.01	0.45
		d_{11}	d_{22}	b_{11}	b_{22}	
ASX 200	Coefficient	0.01	0.50*	0.93*	0.83*	
	p-value	0.96	0.00	0.00	0.00	
MSCI US	Coefficient	0.18	0.80*	0.94*	0.67*	
	p-value	0.20	0.00	0.00	0.00	
MSCI Europe	Coefficient	0.02	0.47*	0.93*	0.88*	
	p-value	0.92	0.00	0.00	0.00	
MSCI Japan	Coefficient	0.11	0.76*	0.94*	0.74*	
	p-value	0.57	0.00	0.00	0.00	
HSI	Coefficient	0.00	0.31**	0.94*	0.88*	
	p-value	1.00	0.04	0.00	0.00	
IBOVESPA	Coefficient	-0.03	0.54*	0.94*	0.85*	
	p-value	0.87	0.00	0.00	0.00	
BSESN	Coefficient	0.06	0.57*	0.94*	0.83*	
	p-value	0.72	0.00	0.00	0.00	
SSECI	Coefficient	0.00	0.00	0.96*	0.86*	
	p-value	1.00	1.00	0.00	0.00	
STI	Coefficient	0.08	0.57*	0.94*	0.84*	
	p-value	0.74	0.00	0.00	0.00	
TAIEX	Coefficient	-0.12	0.45*	0.93*	-0.78*	
	p-value	0.63	0.00	0.00	0.00	
TSXCI	Coefficient	0.15	0.56*	0.94*	0.84*	
	p-value	0.39	0.00	0.00	0.00	

See Note to Table 13.

Table 21 Estimation Results of Variance-Covariance Equations: Hedging with Wheat Futures

		C ₁₁	C ₁₂	C ₂₂	a ₁₁	a ₂₂
ASX 200	Coefficient	4.94	1.21	0.55**	-0.25***	-0.15
	p-value	0.15	0.35	0.02	0.08	0.48
MSCI US	Coefficient	1.74	0.54	0.94*	-0.16***	0.41**
	p-value	0.23	0.62	0.01	0.06	0.03
MSCI Europe	Coefficient	1.62	0.00	0.50**	0.24*	0.14
	p-value	0.12	0.99	0.02	0.01	0.47
MSCI Japan	Coefficient	1.33	0.16	1.24*	0.21**	0.08
	p-value	0.17	0.58	0.00	0.02	0.80
HSI	Coefficient	2.66***	0.55	1.21	0.25**	0.12
	p-value	0.07	0.76	0.37	0.04	0.52
IBOVESPA	Coefficient	15.05*	3.71	4.17***	-0.44*	0.25
	p-value	0.00	0.15	0.06	0.00	0.17
BSESN	Coefficient	1.15	0.00	0.89**	0.18**	0.19
	p-value	0.22	0.99	0.05	0.02	0.33
SSECI	Coefficient	2.17	-0.45	4.37	0.23***	0.30***
	p-value	0.11	0.75	0.36	0.07	0.08
STI	Coefficient	1.58	-0.06	0.77*	0.22**	0.02
	p-value	0.15	0.80	0.01	0.02	0.96
TAIEX	Coefficient	1.83	0.05	1.34	0.21**	0.04
	p-value	0.15	0.81	0.11	0.05	0.90
TSXCI	Coefficient	1.53	0.10	0.49*	-0.23**	0.09
	p-value	0.13	0.63	0.01	0.02	0.81
		d ₁₁	d ₂₂	b ₁₁	b ₂₂	
ASX 200	Coefficient	-0.36**	0.62*	-0.78*	0.86*	
	p-value	0.04	0.00	0.00	0.00	
MSCI US	Coefficient	-0.20	0.82*	-0.93*	0.69*	
	p-value	0.22	0.00	0.00	0.00	
MSCI Europe	Coefficient	0.03	0.46*	0.92*	0.89*	
	p-value	0.92	0.00	0.00	0.00	
MSCI Japan	Coefficient	0.02	0.73*	0.94*	0.75*	
	p-value	0.95	0.00	0.00	0.00	
HSI	Coefficient	-0.24	0.34*	-0.87*	0.89*	
	p-value	0.18	0.01	0.00	0.00	
IBOVESPA	Coefficient	0.03	0.53*	-0.08	0.80*	
	p-value	0.95	0.00	0.92	0.00	
BSESN	Coefficient	0.00	0.52*	0.95*	0.85*	
	p-value	0.99	0.00	0.00	0.00	
SSECI	Coefficient	-0.23	0.18	-0.89*	0.58	
	p-value	0.25	0.66	0.00	0.30	
STI	Coefficient	0.09	0.60*	0.93*	0.83*	
	p-value	0.67	0.00	0.00	0.00	
TAIEX	Coefficient	-0.18	0.43*	0.92*	0.85*	
	p-value	0.39	0.00	0.00	0.00	
TSXCI	Coefficient	0.00	0.65*	0.93*	0.85*	
	p-value	1.00	0.00	0.00	0.00	

See Note to Table 13.

4.3 Optimal Hedge Ratio

4.3.1 Time-varying Optimal Hedge Ratio

After that, we estimate the optimal time-varying hedge ratio in Equation (8) using diagonal BEKK-TGARCH model. Tables 22 to 30 display the time-varying optimal hedge ratios between stock markets and hedge asset futures markets. It is found that copper futures yield the mean optimal hedge ratios against global stock indexes that are the highest among the stock-bitcoin pairs (about 0.38), followed by oil and gold futures (0.31), while wheat futures offer mean optimal hedge ratios that are the lowest among the stock-wheat pairs (0.02).

In particular, the IBOVESPA-Oil pair exhibits the highest mean optimal hedge ratio of 0.62 among all hedged portfolios. This implies that an investor on average can short 0.62 dollar of oil futures for each dollar of IBOVESPA in a long position. In contrast, the TAIEX-wheat pair and the SSECI-wheat pair have the lowest mean optimal hedge ratio of -0.02, suggesting that for every dollar invested in TAIEX and SSECI in a long position, 0.02 dollar should be invested in wheat futures in a long position as well. A negative hedge ratio implies that prices of stocks and wheat futures move in opposite directions on average (negative conditional covariance), thus requiring hedgers to take long positions in both. Many minimum hedge ratios are found to be negative, especially when the hedge assets are agricultural commodity futures.

The time-varying optimal hedge ratios are presented in Figure A3 where the hedge ratios in most cases jumped up in the first quarter of Year 2020 on outbreak of COVID epidemic and then plummeted in the second quarter. The reason is that COVID epidemic initially led an instantaneous and sharp collapse in both stock indexes and hedge asset futures (Figures A1 and A2), together with an upsurge in their covariances and hedge ratios. Over about one quarter, the panic in the markets gradually faded and then prices rebounded, albeit at varying pace for different stock indexes and hedge assets. Their covariances and the hedge ratios accordingly plunged back to levels before the outbreak of COVID.

Table 22 Descriptive Statistics for Time-Varying Optimal Hedge Ratio of Stock-Bitcoin Pairs

	Mean	Median	St. Dev	Minimum	Maximum
ASX 200/bitcoin	0.09	0.08	0.03	0.07	0.21
MSCI US/ bitcoin	0.06	0.06	0.02	0.05	0.13
MSCI Europe/ bitcoin	0.07	0.06	0.03	0.01	0.25
MSCI Japan/ bitcoin	0.07	0.06	0.02	0.05	0.18
HSI/ bitcoin	0.06	0.06	0.01	0.02	0.07
IBOVESPA/ bitcoin	0.08	0.08	0.06	-0.42	0.49
BSESN/ bitcoin	0.05	0.05	0.02	-0.15	0.16
SSECI/ bitcoin	0.04	0.04	0.01	0.00	0.09
STI/bitcoin	0.06	0.06	0.01	0.02	0.08
TAIEX/ bitcoin	0.06	0.06	0.03	-0.21	0.21
TSXCI/bitcoin	0.08	0.08	0.03	0.02	0.33

Table 23 Descriptive Statistics for Time-Varying Optimal Hedge Ratio of Stock-Gold Pairs

	Mean	Median	St. Dev	Minimum	Maximum
ASX 200/gold	0.38	0.30	0.26	0.11	2.08
MSCI US/gold	0.16	0.12	0.30	-0.56	2.09
MSCI Europe/gold	0.35	0.32	0.26	-0.39	1.89
MSCI Japan/gold	0.17	0.15	0.14	-0.18	0.38
HSI/ gold	0.36	0.36	0.22	-0.13	1.03

IBOVESPA gold	0.59	0.59	0.25	-0.26	1.08
BSESN/gold	0.22	0.21	0.07	0.61	0.10
SSECI/gold	0.35	0.34	0.17	-0.12	0.86
STI/gold	0.31	0.30	0.13	0.07	1.14
TAIEX/ gold	0.29	0.27	0.14	0.05	1.27
TSXCI/ gold	0.33	0.27	0.22	0.11	1.82

Table 24 Descriptive Statistics for Time-Varying Optimal Hedge Ratio of Stock-Silver Pairs

	Mean	Median	St. Dev	Minimum	Maximum
ASX 200/silver	0.24	0.22	0.14	0.08	1.04
MSCI US/ silver	0.15	0.11	0.13	0.02	0.97
MSCI Europe/ silver	0.21	0.21	0.09	0.07	0.84
MSCI Japan/ silver	0.12	0.10	0.09	0.00	1.01
HSI/ silver	0.22	0.22	0.13	-0.06	0.52
IBOVESPA/ silver	0.42	0.43	0.19	0.08	1.01
BSESN/ silver	0.11	0.09	0.07	0.02	0.51
SSECI/ silver	0.17	0.15	0.11	0.01	0.54
STI/ silver	0.19	0.18	0.10	0.04	0.70
TAIEX/ silver	0.18	0.18	0.10	-0.09	0.49
TSXCI/silver	0.25	0.24	0.14	0.08	1.08

Table 25 Descriptive Statistics for Time-Varying Optimal Hedge Ratio of Stock-Palladium Pairs

	Mean	Median	St. Dev	Minimum	Maximum
ASX 200/palladium	0.14	0.11	0.07	-0.01	0.47
MCI US/ palladium	0.15	0.13	0.08	-0.09	0.42
MSCI Europe/palladium	0.15	0.14	0.08	-0.09	0.45
MSCI Japan/ palladium	0.13	0.11	0.08	-0.01	0.99
HSI/ palladium	0.17	0.17	0.04	0.05	0.26
IBOVESPA/ palladium	0.24	0.25	0.04	0.07	0.35
BSESN/ palladium	0.12	0.11	0.06	-0.08	0.30
SSECI/ palladium	0.14	0.15	0.06	-0.03	0.26
STI/ palladium	0.13	0.12	0.04	0.31	0.01
TAIEX/ palladium	0.14	0.14	0.03	0.05	0.24
TSXCI/ palladium	0.15	0.13	0.09	-0.02	0.53

Table 26 Descriptive Statistics for Time-Varying Optimal Hedge Ratio of Stock-Copper Pairs

	Mean	Median	St. Dev	Minimum	Maximum
ASX 200/copper	0.39	0.42	0.13	-0.40	1.33
MSCI US/copper	0.28	0.25	0.15	-0.09	1.11
MSCI Europe/copper	0.37	0.40	0.10	0.10	1.24
MSCI Japan/copper	0.27	0.28	0.07	0.03	0.98
HSI/copper	0.50	0.54	0.09	0.17	0.60
IBOVESPA/copper	0.57	0.51	0.22	0.15	2.06
BSESN/copper	0.27	0.23	0.13	0.09	1.21
SSECI/copper	0.44	0.47	0.10	0.09	0.58
STI/copper	0.33	0.35	0.08	0.13	0.68
TAIEX/copper	0.41	0.43	0.09	0.18	0.76
TSXCI/copper	0.37	0.34	0.17	0.15	1.71

Table 27 Descriptive Statistics for Time-Varying Optimal Hedge Ratio of Stock-Oil Pairs

	Mean	Median	St. Dev	Minimum	Maximum
ASX 200/oil	0.36	0.32	0.19	0.07	1.59
MSCI US/oil	0.30	0.25	0.18	0.00	1.19
MSCI Europe/oil	0.32	0.30	0.16	-0.06	1.35
MSCI Japan/oil	0.29	0.29	0.18	0.05	1.82
HSI/oil	0.25	0.28	0.17	-0.29	0.69
IBOVESPA/oil	0.62	0.61	0.23	0.11	1.64
BSESN/oil	0.16	0.14	0.13	-0.07	0.73
SSECI/oil	0.22	0.21	0.09	0.03	0.42
STI/oil	0.26	0.26	0.13	0.03	1.06
TAIEX/oil	0.19	0.19	0.07	0.04	0.47
TSXCI/oil	0.42	0.41	0.15	0.08	1.44

Table 28 Descriptive Statistics for Time-Varying Optimal Hedge Ratio of Stock-Corn Pairs

	Mean	Median	St. Dev	Minimum	Maximum
ASX 200/corn	0.04	0.02	0.09	-0.18	0.60
MSCI US/ corn	0.01	0.01	0.10	-0.3	0.44
MSCI Europe/ corn	0.00	0.01	0.07	-0.23	0.31
MSCI Japan/ corn	0.03	0.03	0.07	-0.33	0.37
HSI/ corn	0.05	0.06	0.08	-0.22	0.27
IBOVESPA/ corn	0.31	0.29	0.18	0.02	1.06
BSESN/ corn	0.00	-0.03	0.16	-0.16	0.95
SSECI/ corn	0.04	0.04	0.05	-0.18	0.22
STI/ corn	0.05	0.02	0.07	-0.08	0.43
TAIEX/ corn	0.00	0.00	0.04	-0.24	0.12
TSXCI/ corn	0.04	0.00	0.12	-0.02	0.95

Table 29 Descriptive Statistics for Time-Varying Optimal Hedge Ratio of Stock-Orange Juice Pairs

	Mean	Median	St. Dev	Minimum	Maximum
ASX 200/orange juice	0.05	0.05	0.08	-0.51	0.40
MSCI US/ orange juice	0.04	0.03	0.07	-0.09	0.76
MSCI Europe/ orange juice	0.04	0.05	0.02	-0.07	0.14
MSCI Japan/ orange juice	0.05	0.03	0.04	0.01	0.48
HSI/ orange juice	0.05	0.05	0.05	-0.10	0.19
IBOVESPA/ orange juice	0.11	0.12	0.04	-0.04	0.21
BSESN/ orange juice	0.07	0.07	0.04	-0.03	0.04
SSECI/ orange juice	0.05	0.05	0.05	-0.08	0.21
STI/ orange juice	0.02	0.01	0.03	-0.01	0.24
TAIEX/ orange juice	0.04	0.04	0.03	-0.07	0.12
TSXCI/ orange juice	0.06	0.06	0.05	-0.03	0.56

Table 30 Descriptive Statistics for Time-Varying Optimal Hedge Ratio of Stock-Wheat Pairs

	Mean	Median	St. Dev	Minimum	Maximum
ASX 200/ wheat	0.03	0.04	0.09	-0.57	0.46
MSCI US/ wheat	0.01	0.02	0.12	-0.86	0.59
MSCI Europe/ wheat	0.01	0.01	0.05	-0.11	0.24
MSCI Japan/wheat	0.04	0.03	0.02	0.00	0.18
HSI/ wheat	0.01	0.02	0.03	-0.11	0.12
IBOVESPA/ wheat	0.18	0.21	0.11	-0.15	0.73

BSESN/ wheat	0.00	0.00	0.04	-0.10	0.12
SSECI/ wheat	-0.02	-0.02	0.04	-0.18	0.24
STI/ wheat	0.01	0.00	0.03	-0.02	0.15
TAIEX/ wheat	-0.02	-0.01	0.03	-0.19	0.02
TSXCI/ wheat	0.02	0.03	0.02	-0.06	0.07

4.3.2. Static Optimal Hedge Ratio

Table 31 reports the static optimal hedge ratios estimated using OLS as shown in Equation (10). Among all, gold futures offer the highest optimal hedge ratio on average, followed by copper and oil futures, while agricultural commodity futures still contribute to the lowest optimal hedge ratios, with several zero and even negative hedge ratios in the case of wheat. Particularly, IBOVESPA-Oil pair has the highest static optimal hedge ratio of 0.75 among all stock/futures pairs under the OLS. MSCI Europe-Wheat Futures pair has the lowest optimal hedge ratio of -0.06.

Table 31. Static Optimal Hedge Ratios

	Bitcoin	Gold	Silver	Palladium	Copper	Oil
ASX 200	0.54	0.60	0.39	0.21	0.09	0.49
MSCI US	0.07	0.41	0.28	0.17	0.38	0.39
MSCI Europe	0.11	0.55	0.32	0.21	0.39	0.29
MSCI Japan	0.08	0.48	0.26	0.19	0.32	0.32
HSI	0.05	0.46	0.21	0.14	0.46	0.23
IBOVESPA	0.13	0.65	0.51	0.29	0.68	0.75
BSESN	0.07	0.31	0.22	0.10	0.30	0.23
SSECI	0.04	0.37	0.19	0.11	0.39	0.20
STI	0.07	0.37	0.22	0.15	0.37	0.28
TAIEX	0.06	0.40	0.24	0.15	0.40	0.24
TSXCI	0.11	0.61	0.38	0.22	0.50	0.50
Average	0.12	0.47	0.29	0.18	0.39	0.36
	Corn	Orange Juice	Wheat			
ASX 200	0.10	0.04	0.05			
MSCI US	0.04	0.04	0.00			
MSCI Europe	0.01	0.06	-0.06			
MSCI Japan	0.03	0.07	0.03			
HSI	0.04	0.05	-0.01			
IBOVESPA	0.29	0.16	0.16			
BSESN	0.03	0.05	-0.05			
SSECI	0.04	0.07	0.00			
STI	0.07	0.02	-0.01			
TAIEX	0.04	0.03	0.00			
TSXCI	0.10	0.07	0.03			
Average	0.07	0.06	0.01			

4.4 Hedging Effectiveness

Hedging effectiveness (Equation 11) is estimated by diagonal BEKK-TGARCH and OLS. The results are presented in the following Tables 32-49. It is found that the hedged TSXCI-Oil portfolio provides the highest hedging effectiveness of 41% under GARCH. The hedged STI-Orange portfolio, however, yields the lowest hedging effectiveness of -2% under GARCH. The

negative hedging effectiveness indicates that the variance of the hedged portfolio is even larger than that of the unhedged portfolio.

Table 32 Hedging Effectiveness of Bitcoin Futures estimated by diagonal BEKK-TGARCH

	ASX 200	MSCI US	MSCI Europe	MSCI Japan	HSI	IBOVESPA
Variance Unhedged	10.13	8.43	7.89	7.38	9.05	27.19
Variance hedged	9.23	7.94	7.76	6.85	8.73	23.61
Hedging Effectiveness	0.09	0.06	0.15	0.07	0.04	0.13
	BSESN	SSECI	STI	TAIEX	TSXCI	
Variance Unhedged	8.36	7.28	6.29	7.51	8.94	
Variance hedged	7.56	7.07	5.75	6.75	8.00	
Hedging Effectiveness	0.10	0.03	0.09	0.10	0.11	

Table 33. Hedging Effectiveness of Bitcoin Futures estimated by OLS

	ASX 200	MSCI US	MSCI Europe	MSCI Japan	HSI	IBOVESPA
Variance Unhedged	10.13	8.43	9.15	7.38	9.05	27.19
Variance hedged	9.06	7.90	7.98	6.74	8.76	25.45
Hedging Effectiveness	0.11	0.06	0.13	0.09	0.03	0.06
	BSESN	SSECI	STI	TAIEX	TSXCI	
Variance Unhedged	8.36	7.28	6.29	7.51	8.94	
Variance hedged	7.91	7.10	5.78	7.14	7.77	
Hedging Effectiveness	0.05	0.02	0.08	0.05	0.13	

Table 34 Hedging Effectiveness of Gold Futures estimated by diagonal BEKK-TGARCH

	ASX 200	MSCI US	MSCI Europe	MSCI Japan	HSI	IBOVESPA
Variance Unhedged	10.13	8.43	7.89	7.38	9.05	27.19
Variance hedged	8.75	7.88	7.66	5.79	8.11	25.06
Hedging Effectiveness	0.14	0.07	0.16	0.21	0.10	0.08
	BSESN	SSECI	STI	TAIEX	TSXCI	
Variance Unhedged	8.36	7.28	6.29	7.51	8.94	

Variance hedged	8.09	6.62	5.51	6.77	7.07	
Hedging Effectiveness	0.03	0.09	0.12	0.10	0.21	

Table 35 Hedging Effectiveness of Gold Futures estimated by OLS

	ASX 200	MSCI US	MSCI Europe	MSCI Japan	HSI	IBOVESPA
Variance Unhedged	10.13	8.43	9.15	7.38	9.05	27.19
Variance hedged	8.67	7.74	7.90	6.42	8.41	25.49
Hedging Effectiveness	0.14	0.08	0.14	0.13	0.07	0.06
	BSESN	SSECI	STI	TAIEX	TSXCI	
Variance Unhedged	8.36	7.28	6.29	7.51	8.94	
Variance hedged	7.98	6.71	5.74	6.87	7.41	
Hedging Effectiveness	0.05	0.08	0.09	0.09	0.17	

Table 36 Hedging Effectiveness of Silver Futures estimated by diagonal BEKK-TGARCH

	ASX 200	MSCI US	MSCI Europe	MSCI Japan	HSI	IBOVESPA
Variance Unhedged	10.13	8.43	7.89	7.38	9.05	27.19
Variance hedged	6.50	6.40	7.07	5.91	8.10	21.74
Hedging Effectiveness	0.36	0.24	0.23	0.20	0.10	0.20
	BSESN	SSECI	STI	TAIEX	TSXCI	
Variance Unhedged	8.36	7.28	6.29	7.51	8.94	
Variance hedged	7.56	6.63	4.78	6.58	5.44	
Hedging Effectiveness	0.10	0.09	0.24	0.12	0.39	

Table 37 Hedging Effectiveness of Silver Futures estimated by OLS

	ASX 200	MSCI US	MSCI Europe	MSCI Japan	HSI	IBOVESPA
Variance Unhedged	10.13	8.43	9.15	7.38	9.05	27.19
Variance hedged	7.54	7.10	7.46	6.23	8.28	22.79
Hedging Effectiveness	0.26	0.16	0.19	0.16	0.09	0.16

	BSESN	SSECI	STI	TAIEX	TSXCI	
Variance Unhedged	8.36	7.28	6.29	7.51	8.94	
Variance hedged	7.57	6.66	5.45	6.56	6.46	
Hedging Effectiveness	0.09	0.08	0.13	0.13	0.28	

Table 38 Hedging Effectiveness of Palladium Futures estimated by diagonal BEKK-TGARCH

	ASX 200	MSCI US	MSCI Europe	MSCI Japan	HSI	IBOVESPA
Variance Unhedged	10.13	8.43	7.89	7.38	9.05	27.19
Variance hedged	8.62	6.24	6.80	5.38	8.06	23.82
Hedging Effectiveness	0.15	0.13	0.26	0.27	0.11	0.12
	BSESN	SSECI	STI	TAIEX	TSXCI	
Variance Unhedged	8.36	7.28	6.29	7.51	8.94	
Variance hedged	7.76	6.73	5.36	6.60	7.06	
Hedging Effectiveness	0.07	0.08	0.15	0.12	0.21	

Table 39 Hedging Effectiveness of Palladium Futures estimated by OLS

	ASX 200	MSCI US	MSCI Europe	MSCI Japan	HSI	IBOVESPA
Variance Unhedged	10.13	8.43	9.15	7.38	9.05	27.19
Variance hedged	8.46	7.39	7.43	5.93	8.27	23.93
Hedging Effectiveness	0.17	0.12	0.19	0.20	0.09	0.12
	BSESN	SSECI	STI	TAIEX	TSXCI	
Variance Unhedged	8.36	7.28	6.29	7.51	8.94	
Variance hedged	7.96	6.83	5.45	6.67	7.07	
Hedging Effectiveness	0.05	0.06	0.13	0.11	0.21	

Table 40 Hedging Effectiveness of Copper Futures estimated by diagonal BEKK-TGARCH

	ASX 200	MSCI US	MSCI Europe	MSCI Japan	HSI	IBOVESPA
Variance Unhedged	10.13	8.43	7.89	7.38	9.05	27.19

Variance hedged	6.57	6.31	9.15	6.74	6.85	21.02
Hedging Effectiveness	0.35	0.25	0.13	0.09	0.24	0.23
	BSESN	SSECI	STI	TAIEX	TSXCI	
Variance Unhedged	8.36	7.28	6.29	7.51	8.94	
Variance hedged	6.99	5.55	4.73	5.82	5.57	
Hedging Effectiveness	0.16	0.24	0.25	0.23	0.38	

Table 41 Hedging Effectiveness of Copper Futures estimated by OLS

	ASX 200	MSCI US	MSCI Europe	MSCI Japan	HSI	IBOVESPA
Variance Unhedged	10.13	8.43	9.15	7.38	9.05	27.19
Variance hedged	7.28	7.00	7.60	6.36	6.96	22.61
Hedging Effectiveness	0.28	0.17	0.17	0.14	0.23	0.17
	BSESN	SSECI	STI	TAIEX	TSXCI	
Variance Unhedged	8.36	7.28	6.29	7.51	8.94	
Variance hedged	7.47	5.75	4.96	5.94	6.44	
Hedging Effectiveness	0.11	0.21	0.21	0.21	0.28	

Table 42 Hedging Effectiveness of Oil Futures by diagonal BEKK-TGARCH

	ASX 200	MSCI US	MSCI Europe	MSCI Japan	HSI	IBOVESPA
Variance Unhedged	10.13	8.43	7.89	7.38	9.05	27.19
Variance hedged	6.26	5.83	8.40	7.99	9.00	19.18
Hedging Effectiveness	0.38	0.31	0.08	-0.08	0.01	0.29
	BSESN	SSECI	STI	TAIEX	TSXCI	
Variance Unhedged	8.36	7.28	6.29	7.51	8.94	
Variance hedged	7.03	6.75	4.97	6.67	5.27	
Hedging Effectiveness	0.16	0.07	0.21	0.11	0.41	

Table 43 Hedging Effectiveness of Oil Futures estimated by OLS

	ASX 200	MSCI US	MSCI Europe	MSCI Japan	HSI	IBOVESPA
Variance Unhedged	10.13	8.43	9.15	7.38	9.05	27.19
Variance hedged	7.23	6.65	8.10	6.20	8.43	20.42
Hedging Effectiveness	0.29	0.21	0.12	0.16	0.07	0.25
	BESN	SSECI	STI	TAIEX	TSXCI	
Variance Unhedged	8.36	7.28	6.29	7.51	8.94	
Variance hedged	7.75	6.78	5.34	6.81	5.97	
Hedging Effectiveness	0.07	0.07	0.15	0.09	0.33	

Table 44 Hedging Effectiveness of Corn Futures estimated by diagonal BEKK-TGARCH

	S&P/ASX 200	MSCI US	MSCI Europe	MSCI Japan	HSI	IBOVESPA
Variance Unhedged	10.13	8.43	7.89	7.38	9.05	27.19
Variance hedged	9.52	8.04	8.93	7.26	9.00	24.82
Hedging Effectiveness	0.06	0.05	0.02	0.02	0.01	0.09
	BESN	TSXCI	STI	SSECI	TAIEX	
Variance Unhedged	8.36	8.94	6.29	7.28	7.51	
Variance hedged	7.64	8.04	5.95	7.26	7.45	
Hedging Effectiveness	0.09	0.10	0.05	0.00	0.01	

Table 45 Hedging Effectiveness of Corn Futures estimated by OLS

	ASX 200	MSCI US	MSCI Europe	MSCI Japan	HSI	IBOVESPA
Variance Unhedged	10.13	8.43	9.15	7.38	9.05	27.19
Variance hedged	10.01	8.40	9.15	7.36	9.03	25.78
Hedging Effectiveness	0.01	0.00	0.00	0.00	0.00	0.05
	BESN	SSECI	STI	TAIEX	TSXCI	
Variance Unhedged	8.36	7.28	6.29	7.51	8.94	
Variance hedged	8.35	7.25	6.22	7.49	8.79	

Hedging Effectiveness	0.00	0.00	0.01	0.00	0.02	
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Table 46 Hedging Effectiveness of Orange Juice Futures estimated by diagonal BEKK-TGARCH

	ASX 200	MSCI US	MSCI Europe	MSCI Japan	HSI	IBOVESPA
Variance Unhedged	10.13	8.43	7.89	7.38	9.05	27.19
Variance hedged	9.14	8.34	8.97	7.99	8.85	27.00
Hedging Effectiveness	0.10	0.01	0.02	0.03	0.02	0.01
	BESN	SSECI	STI	TAIEX	TSXCI	
Variance Unhedged	8.36	7.28	6.29	7.51	8.94	
Variance hedged	8.43	7.38	6.40	7.51	8.79	
Hedging Effectiveness	-0.01	-0.01	-0.02	0.01	0.02	

Table 47 Hedging Effectiveness of Orange Juice Futures estimated by OLS

	ASX 200	MSCI US	MSCI Europe	MSCI Japan	HSI	IBOVESPA
Variance Unhedged	10.13	8.43	9.15	7.38	9.05	27.19
Variance hedged	10.10	8.41	9.09	7.29	9.01	26.69
Hedging Effectiveness	0.00	0.00	0.01	0.01	0.01	0.02
	BESN	SSECI	STI	TAIEX	TSXCI	
Variance Unhedged	8.36	7.28	6.29	7.51	8.94	
Variance hedged	8.31	7.19	6.28	7.49	8.84	
Hedging Effectiveness	0.01	0.01	0.00	0.00	0.01	

Table 48 Hedging Effectiveness of Wheat Futures estimated by diagonal BEKK-TGARCH

	ASX 200	MSCI US	MSCI Europe	MSCI Japan	HSI	IBOVESPA
Variance Unhedged	10.13	8.43	7.89	7.38	9.05	27.19
Variance hedged	9.46	7.81	8.87	7.28	9.04	26.24
Hedging Effectiveness	0.07	0.07	0.03	0.01	0.00	0.04

	BESN	SSECI	STI	TAIEX	TSXCI	
Variance Unhedged	8.36	7.28	6.29	7.51	8.94	
Variance hedged	8.28	7.21	6.30	7.54	8.94	
Hedging Effectiveness	0.01	0.01	0.00	0.00	0.00	

Table 49 Hedging Effectiveness of Wheat Futures estimated by OLS

	ASX 200	MSCI US	MSCI Europe	MSCI Japan	HSI	IBOVESPA
Variance Unhedged	10.13	8.43	9.15	7.38	9.05	27.19
Variance hedged	10.06	8.43	9.05	7.34	9.05	26.56
Hedging Effectiveness	0.01	0.00	0.01	0.00	0.00	0.02
	BESN	SSECI	STI	TAIEX	TSXCI	
Variance Unhedged	8.36	7.28	6.29	7.51	8.94	
Variance hedged	8.29	7.28	6.29	7.51	8.92	
Hedging Effectiveness	0.01	0.00	0.00	0.00	0.00	

4.5. Comparison of Hedging Effectiveness in Different Dimensions

We compare the hedging performance across various hedge asset futures, stock indexes and estimation methods by compiling the data from Tables 36-49 in Section 4.4.

Table 50 shows that copper futures provide the highest hedging effectiveness of over 20% on average, followed by silver futures, oil futures, palladium futures, gold futures, bitcoin futures, corn futures, orange juice and wheat futures. It can be seen that agricultural commodities provide average hedging effectiveness of less than 5% only. Bitcoin is also not a favorite hedge asset, and it is consistent with the findings of for example, Baur et al. (2022) and Corbet et al. (2020) though this is in contrast to Dyhrberg (2016) and Kliber et al. (2019).

Table 50. Comparison of Hedging Effectiveness among **Different Hedge Assets**

	Bitcoin		Gold		Silver	
	GARCH	OLS	GARCH	OLS	GARCH	OLS
ASX 200	0.09	0.11	0.14	0.14	0.36	0.26
MSCI US	0.06	0.06	0.07	0.08	0.24	0.16
MSCI Europe	0.15	0.13	0.16	0.14	0.23	0.19
MSCI Japan	0.07	0.09	0.21	0.13	0.20	0.16
HSI	0.04	0.03	0.10	0.07	0.10	0.09
IBOVESPA	0.13	0.06	0.08	0.06	0.20	0.16
BESN	0.10	0.05	0.03	0.05	0.10	0.09
TAIEX	0.10	0.05	0.10	0.09	0.12	0.13

SSECI	0.03	0.02	0.09	0.08	0.09	0.08
STI	0.09	0.08	0.12	0.09	0.24	0.13
TSXCI	0.11	0.13	0.21	0.17	0.39	0.28
Average	0.09	0.07	0.12	0.10	0.21	0.16
	Palladium		Copper		Crude oil	
	GARCH	OLS	GARCH	OLS	GARCH	OLS
ASX 200	0.15	0.17	0.35	0.28	0.38	0.29
MSCI US	0.13	0.12	0.25	0.17	0.31	0.21
MSCI Europe	0.26	0.19	0.13	0.17	0.08	0.12
MSCI Japan	0.27	0.20	0.09	0.14	-0.08	0.16
HSI	0.11	0.09	0.24	0.23	0.01	0.07
IBOVESPA	0.12	0.12	0.23	0.17	0.29	0.25
BSESN	0.07	0.05	0.16	0.11	0.16	0.07
SSECI	0.08	0.06	0.24	0.21	0.07	0.07
STI	0.15	0.13	0.25	0.21	0.21	0.15
TAIEX	0.12	0.11	0.23	0.21	0.11	0.09
TSXCI	0.21	0.21	0.38	0.28	0.41	0.33
Average	0.15	0.13	0.23	0.20	0.18	0.16
	Corn		Orange Juice		Wheat	
	GARCH	OLS	GARCH	OLS	GARCH	OLS
ASX 200	0.06	0.01	0.10	0.00	0.07	0.01
MSCI US	0.05	0.00	0.01	0.00	0.07	0.00
MSCI Europe	0.02	0.00	0.02	0.01	0.03	0.01
MSCI Japan	0.02	0.00	0.03	0.01	0.01	0.00
HSI	0.01	0.00	0.02	0.01	0.00	0.00
IBOVESPA	0.09	0.05	0.01	0.02	0.04	0.02
BSESN	0.09	0.00	-0.01	0.01	0.01	0.01
SSECI	0.00	0.00	-0.01	0.01	0.01	0.00
STI	0.05	0.01	-0.02	0.00	0.00	0.00
TAIEX	0.01	0.00	0.01	0.00	0.00	0.00
TSXCI	0.10	0.02	0.02	0.01	0.00	0.00
Average	0.04	0.01	0.02	0.01	0.02	0.01

From Table 51, ASX 200 and TSXCI are on average hedged most effectively with hedging effectiveness of about 20% on average. Particularly, hedging TSXCI with oil (silver) can achieve hedging effectiveness of 0.41 (0.39) when hedging ASX200 with oil (silver) can enjoy the hedging effectiveness of 0.38 (0.36). The next best hedging performance is achieved in MSCI US, IBOVESPA, STI and MSCI Europe, while others with weak hedging performance such as HSI, BSESN, SSECI and TAIEX, have average hedging effectiveness of 10% or less only.

Table 51 Comparison of Hedging Effectiveness among **Different Stock Indexes**

	ASX 200		MSCI US		MSCI Europe		MSCI Japan	
	GARCH	OLS	GARCH	OLS	GARCH	OLS	GARCH	OLS
Bitcoin	0.09	0.11	0.06	0.06	0.15	0.13	0.07	0.09
Gold	0.14	0.14	0.07	0.08	0.16	0.14	0.21	0.13
Silver	0.36	0.26	0.24	0.16	0.23	0.19	0.20	0.16
Palladium	0.15	0.17	0.13	0.12	0.26	0.19	0.27	0.20
Copper	0.35	0.28	0.25	0.17	0.13	0.17	0.09	0.14
Crude oil	0.38	0.29	0.31	0.21	0.08	0.12	-0.08	0.16
Corn	0.06	0.01	0.05	0.00	0.02	0.00	0.02	0.00
Orange juice	0.10	0.00	0.01	0.00	0.02	0.01	0.03	0.01
Wheat	0.07	0.01	0.07	0.00	0.03	0.01	0.01	0.00
Average	0.21	0.16	0.15	0.10	0.13	0.12	0.10	0.11
	HSI		IBOVESPA		BSESN		SSECI	
	GARCH	OLS	GARCH	OLS	GARCH	OLS	GARCH	OLS
Bitcoin	0.04	0.03	0.13	0.06	0.10	0.05	0.03	0.02
Gold	0.10	0.07	0.08	0.06	0.03	0.05	0.09	0.08
Silver	0.10	0.09	0.20	0.16	0.10	0.09	0.09	0.08
Palladium	0.11	0.09	0.12	0.12	0.07	0.05	0.08	0.06
Copper	0.24	0.23	0.23	0.17	0.16	0.11	0.24	0.21
Crude oil	0.01	0.07	0.29	0.25	0.16	0.07	0.07	0.07
Corn	0.01	0.00	0.09	0.05	0.09	0.00	0.00	0.00
Orange juice	0.02	0.01	0.01	0.02	-0.01	0.01	-0.01	0.01
Wheat	0.00	0.00	0.04	0.02	0.01	0.01	0.01	0.00
Average	0.08	0.07	0.14	0.11	0.09	0.06	0.07	0.07
	STI		TAIEX		TSXCI			
	GARCH	OLS	GARCH	OLS	GARCH	OLS		
Bitcoin	0.09	0.08	0.10	0.05	0.11	0.13		
Gold	0.12	0.09	0.10	0.09	0.21	0.17		
Silver	0.24	0.13	0.12	0.13	0.39	0.28		
Palladium	0.15	0.13	0.12	0.11	0.21	0.21		
Copper	0.25	0.21	0.23	0.21	0.38	0.28		
Crude oil	0.21	0.15	0.11	0.09	0.41	0.33		
Corn	0.05	0.01	0.01	0.00	0.10	0.02		
Orange juice	-0.02	0.00	0.01	0.00	0.02	0.01		
Wheat	0.00	0.00	0.00	0.00	0.00	0.00		
Average	0.14	0.10	0.10	0.09	0.21	0.18		

Table 52 shows that except MSCI Japan, the average hedging effectiveness estimated by diagonal BEKK-TGARCH is higher than that estimated by OLS. The reason is that GARCH produces time-varying hedge ratios that can better capture the fluctuations of conditional variances of returns that static ones cannot.

To summarize the above results, for hedging purposes, we suggest using copper, precious metals and crude oil as hedge assets, rather than agricultural commodities. Time-varying hedge ratios are better estimated by using GARCH than static models using OLS. Stock markets that are effectively hedged include Australia, Canada, US and Europe. Hong Kong, Mainland China, Taiwan and India are however not effectively hedged as per the tests conducted.

5. Conclusion

In the real world, investors often use hedging to reduce investment risk. This paper aims to find out the effectiveness of bitcoin, precious metals, copper crude oil and agricultural futures in hedging against global stock markets. Among global stock indexes, the most representative stock markets are Australia, US, Europe, Japan Brazil, India, Hong Kong, Taiwan, Mainland China, Singapore and Canada. We found that hedging is relatively more effective in Australia and Canada.

To estimate hedge ratios and hedging effectiveness, this paper adopts the Diagonal-BEKK-GARCH model and the traditional OLS method. In contrast to OLS, GARCH models tend to produce more flexible time-varying and more precise estimates of optimal hedge ratios. Thus GARCH is better than OLS for estimating hedge ratios. Moreover, under the GARCH model, estimation of mean and variance equations can help examine the evidence of returns and volatility spillovers between stock markets and futures markets of hedging instruments. We found weak evidence of returns spillovers but strong evidence of volatility spillover. Also, strong evidence of asymmetric leverage effect is found in the conditional variance of stock markets but not in futures markets.

Among hedge assets, the stock-copper hedged portfolio can help reduce the variance of the portfolio the most on average, followed by stock-silver and stock-oil pairs. Hedging with agricultural commodity futures offers little risk reduction and they should not be used as hedge assets.

Our study is related to the practice of decision sciences (Chang, et al., 2018; Hasan-Zadeh, 2019; Tuan, et al., 2022) when our research results can provide investors and portfolio managers with a more comprehensive picture of hedging performance of different hedge assets in global stock markets. These are expected to be useful for making optimal hedging decisions, i.e. selection of hedging strategies and hedging tools.

Table 52 Comparison of Hedging Effectiveness between **Different Methods**

GARCH	ASX 200	MSCI US	MSCI Europe	MSCI Japan	HSI	IBOVESPA	BSESN	SSECI	STI	TAIEX	TSXCI
Bitcoin	0.09	0.06	0.15	0.07	0.04	0.13	0.10	0.03	0.09	0.10	0.11
Gold	0.14	0.07	0.16	0.21	0.10	0.08	0.03	0.09	0.12	0.10	0.21
Silver	0.36	0.24	0.23	0.20	0.10	0.20	0.10	0.09	0.24	0.12	0.39
Palladium	0.15	0.13	0.26	0.27	0.11	0.12	0.07	0.15	0.15	0.12	0.21
Copper	0.35	0.25	0.13	0.09	0.24	0.23	0.16	0.24	0.25	0.23	0.38
Crude oil	0.38	0.31	0.08	-0.08	0.01	0.29	0.16	0.07	0.21	0.11	0.41
Corn	0.06	0.05	0.02	0.02	0.01	0.09	0.09	0.00	0.05	0.01	0.10
Orange juice	0.10	0.01	0.02	0.03	0.02	0.01	-0.01	-0.01	-0.02	0.01	0.02
Wheat	0.07	0.07	0.03	0.01	0.00	0.04	0.01	0.01	0.00	0.00	0.00
Average	0.21	0.15	0.13	0.10	0.08	0.14	0.09	0.08	0.14	0.10	0.21
OLS	ASX 200	MSCI US	MSCI Europe	MSCI Japan	HSI	IBOVESPA	BSESN	SSECI	STI	TAIEX	TSXCI
Bitcoin	0.11	0.06	0.13	0.09	0.03	0.06	0.05	0.02	0.08	0.05	0.13
Gold	0.14	0.08	0.14	0.13	0.07	0.06	0.05	0.08	0.09	0.09	0.17
Silver	0.26	0.16	0.19	0.16	0.09	0.16	0.09	0.08	0.13	0.13	0.28
Palladium	0.17	0.12	0.19	0.20	0.09	0.12	0.05	0.06	0.13	0.11	0.21
Copper	0.28	0.17	0.17	0.14	0.23	0.17	0.11	0.21	0.21	0.21	0.28
Crude oil	0.29	0.21	0.12	0.16	0.07	0.25	0.07	0.07	0.15	0.09	0.33
Corn	0.01	0.00	0.00	0.00	0.00	0.05	0.00	0.00	0.01	0.00	0.02
Orange juice	0.00	0.00	0.01	0.01	0.01	0.02	0.01	0.01	0.00	0.00	0.01
Wheat	0.01	0.00	0.01	0.00	0.00	0.02	0.01	0.00	0.00	0.00	0.00
Average	0.16	0.10	0.12	0.11	0.07	0.11	0.06	0.07	0.10	0.09	0.18

Reference

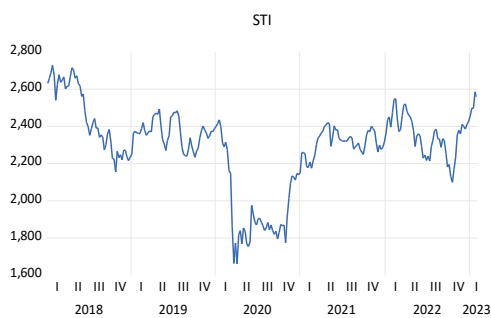
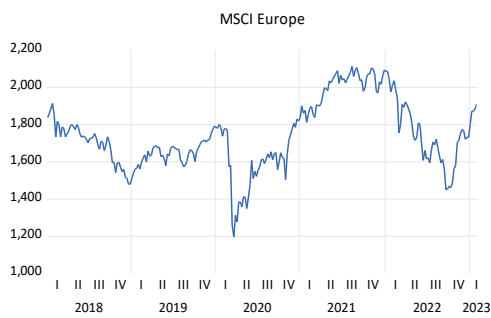
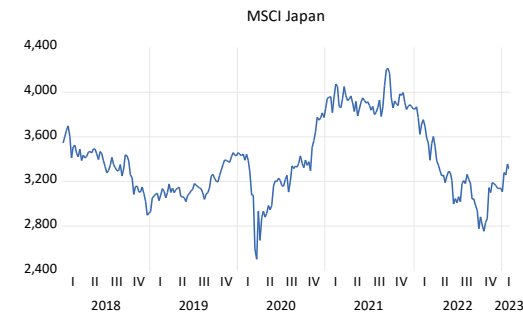
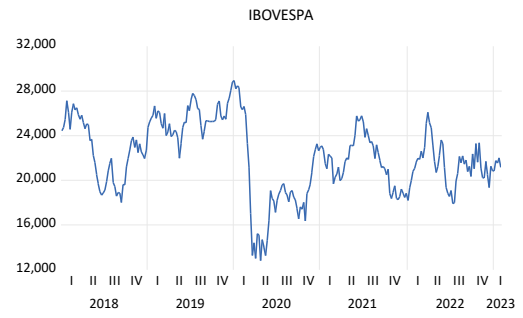
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Appendix

Figure A1 Price Trends of All Stock Indexes under Study



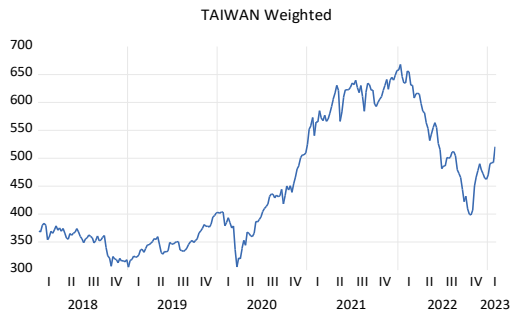
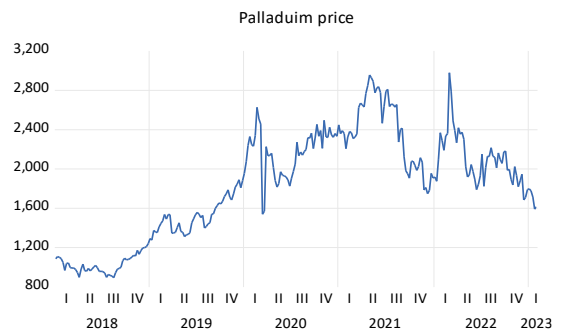
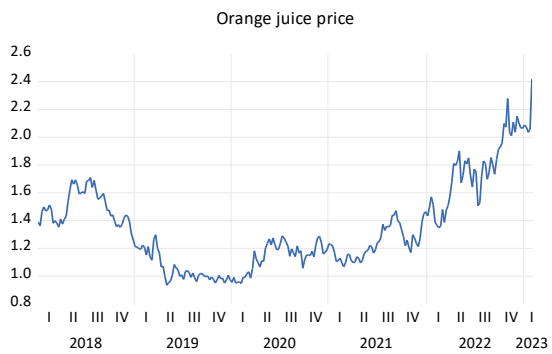


Figure A2 Price Trends of All Futures under Study



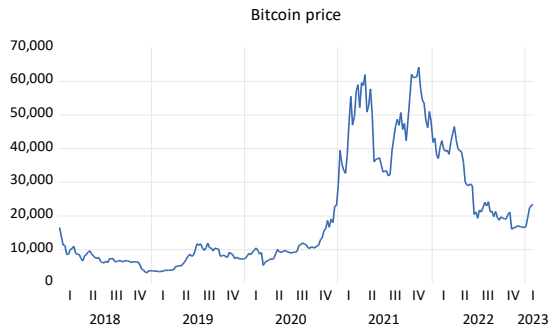
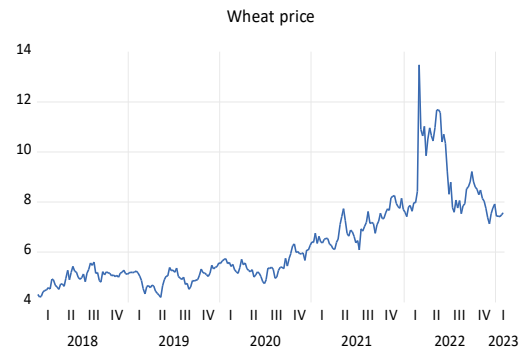


Figure A3 Time-Varying Optimal Hedge Ratios of Bitcoin, Crude oil, Precious and Industrial Metals and Agricultural Futures against Global Stock Indexes

