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Threshold adjustments to intranational PPP in Canada

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Abstract

We study the intranational purchasing power parity (PPP) between 17 pairs of Canadian cities for the period 1984 to 2010 using multivariate tests of threshold cointegration in a threshold vector error correction model. Our results confirm evidence of nonlinear mean reversion in deviations from the PPP in 13 bivariate systems of price indices. Symmetric and asymmetric adjustment processes toward equilibrium are identified in different city pairs. Also, different directions of long-run Granger causality between price indices are found. In Monte Carlo simulations, we estimate the mean bias and the unconditional half-lives for nonlinear PPP deviations. In addition, we measure the absorption rate of shocks and find that different types of shocks are absorbed at different rates by different variables in the systems. This evidence reveals that there are complex nonlinearities in the data.

Keywords: multivariate threshold cointegration; nonlinear adjustments; nonlinear impulse responses; mean bias; unconditional half-life; absorption of shocks

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1. Introduction

Purchasing power parity (PPP) states that international price levels should be the same across every country after they are converted to a common currency. The rationale behind the PPP theory requires perfect commodity arbitrage where spatial traders profit by transporting tradable goods among different countries in the short run. Eventually, price differences are exhausted and the PPP will hold in the long run. Most studies on the PPP hypothesis have focused on international price comparisons. Recently, a growing number of studies have examined the PPP hypothesis using intranational price data. It may be reasonable to argue that intranational PPP should be easier and quicker to achieve than international PPP for the following reasons: fewer trade obstacles, comparable consumer preferences, more integrated internal markets, no exchange rate volatility, and more homogenous aggregate price indices collected by the same statistical institution within a country (Carrion-I-Silvestre et al., 2004). However, any evidence of intranational PPP violations will have important policy implications because persistent intranational PPP deviations may imply the possibility of persistent regional development imbalances, labour immobility, productivity differentials, and market segmentation among different locations inside a country. These may lead to misallocation of resources. Furthermore, it is argued that static and dynamic gains achieved by comparative advantage, economies of scale, diffusion of technical knowledge, and efficiency from competition cannot be fully exploited if intranational PPP is not achieved.

Canada is a well-developed market economy and the second largest country in the world by total land area: its lands stretch from the Pacific Ocean in the west to the

Atlantic Ocean in the east and northward to the Arctic Ocean. Given this distinctive geographical span, Canada is an ideal country for the study of intranational PPP. On one hand, Canada has a well-established highway network that connects the entire country, transportation costs is likely reduced. Also, interprovincial trade barriers among the different provinces of the country have been diminishing since the first interprovincial trade agreement, the Agreement on Internal Trade (AIT), took effect in 1995.[1] Furthermore, with the help of some federal government policies,[2] most factor inputs are comparatively mobile and the overall regional balance has increased. On the other hand, however, it can be argued that given its huge land size together with salient differences in agro-climate conditions, provincial governance, internal trade policies,[3] and local tax systems, arbitrage activities may be hindered to a certain degree and intranational PPP may not occur in all cities in Canada.[4]

In testing for intranational PPP, transaction costs should not be ignored, although they are expected to be much smaller inside a country than between countries, because the market frictions arising from transaction costs will impede the operation of goods arbitrage when the potential gains from arbitrage do not outweigh these costs. As pointed out by Heckscher (1916), the presence of transaction costs will create a neutral band or 'band of inaction' inside which the relative prices are too small to

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[1] The 1995 AIT specifies the broad principles applied to any provincial use of trade-restricting measures in Canada. These include nondiscrimination and rights of entry to or exit from provincial markets. The purpose of the AIT is to reduce and/or eliminate, as far as possible, barriers to the free movement of individuals, goods, services, and investment within Canada and to establish an open, efficient, and stable domestic market (http://www.ait-aci.ca).

[2] For example, the equalization payments policy is implemented for transferring tax money from the richer provinces to the poorer provinces (http://www.fin.gc.ca/fedprov/eqp-eng.asp).

[3] There are three levels of government in Canada: federal, provincial, and municipal. Even though most trade policies are established at the federal level, each province may have its own unique trade policies (e.g. Trade Assistance Programs and Services in New Brunswick; Trade Team British Columbia in British Columbia) which may, in turn, result in market segmentation.

[4] According to the Canadian Real Estate Association (2011), average house prices in different regions are substantially different. For example, the price of the same type of house can be three to four times higher in Vancouver, British Columbia than in Saint John, New Brunswick (http://www.crea.ca).

induce arbitrage and then the deviations from PPP would be non-mean reverting. In cases where the price differences are large enough to exceed the arbitrage costs outside the band, arbitrage activities take place and the deviations from PPP will switch to become mean reverting. The resulting adjustment process of related price series is therefore nonlinear and discontinuous (Dumas, 1992; Sercu et al., 1995). As the data on transaction costs are typically unobservable and the nonlinear nature of the adjustment process as predicted by the transaction cost model of goods arbitrage can be adequately captured by a threshold autoregressive (TAR) model (Obstfeld and Taylor, 1997), previous studies (e.g., Goodwin and Piggott, 2001; Juvenal and Taylor, 2008) have adopted the two-step threshold cointegration approach suggested by Balke and Fomby (1997) to test for the price convergence of individual goods between distant markets under a univariate TAR framework. This approach is based on price data alone and also it can explicitly take into account the threshold effects of price adjustments toward long-run equilibrium.

In the current paper, we apply the threshold cointegrating approach to test the existence of intranational PPP in 17 pairs of Canadian cities and attempt to uncover the potential nonlinearities and asymmetries in the adjustments of PPP deviations.[5] We follow the same two-step testing strategy as that used in Balke and Fomby (1997) but adopt the multivariate tests of cointegration and threshold nonlinearity in a threshold vector error correction model (TVECM). We also employ the model specification tests to impose parametric restrictions on the TVECM that are consistent with the transaction-cost theory of goods arbitrage.

[5] Previous studies of intranational PPP have typically adopted panel methods to test for unit roots in the relative prices (e.g., Cecchetti et al., 2002; Ceglowski, 2003). However, the panel methods assume a linear process and fail to exploit the nonlinear adjustment dynamics in the presence of transaction costs. The parameter estimates under the linear framework may also suffer from a linear specification bias when the true adjustment dynamics are nonlinear in nature (Taylor, 2001).

In addition, cointegration in the context of PPP implies that at least one price variable adjusts in response to the last period's PPP deviation under an error correction mechanism. Hence, we test the statistical significance of the error-correction coefficients in the TVECM in order to examine the directions of the Granger causality between the prices under study and identify the sources of the adjustments toward the PPP equilibrium. Moreover, the long-run PPP actually reflects a certain degree of PPP persistence and the speed of convergence between geographically separated locations. Many previous studies of PPP persistence in nonlinear models have calculated regime-specific or conditional half-life estimates. We estimate the full-sample, unconditional half-life of nonlinear PPP deviations in the TVECM using the simulation techniques suggested by Lo (2008). Further, van Dijk et al. (2007) proposed using different measures of absorption to investigate how fast shocks are absorbed in multivariate nonlinear models. We estimate the absorption rates of shocks in the TVECM in order to understand more about the asymmetric propagation of shocks to the PPP equilibrium. These measures of persistence and absorption can convey information about different aspects of the propagation of shocks in a dynamic system.

The rest of the paper is organized as follows: Section 2 explains the econometric methodologies employed; the data and empirical results are described in Section 3; and concluding remarks are made in Section 4.

2. Econometric Methodology

2.1 Threshold vector error correction models

A three-regime (two thresholds) TVECM is suggested to be the most suitable model that satisfies the economic requirements for the analysis of the bidirectional price adjustments in the presence of transaction costs (Meyer, 2004).[6] The unrestricted bivariate three-regime TVECM for price indices X_t defined as $(P_{1t}, P_{2t})'$, where P_{1t} and P_{2t} are the logarithms of the price indices at time t in two locations, with lag length ℓ is written as follows:

$$\Delta X_t = \begin{cases} \mu_1 + \alpha_1 Z_{t-1} + \sum_{h=1}^{\ell} \Phi_{1h} \Delta X_{t-h} + \varepsilon_t & \text{if } Z_{t-1} \le \gamma_1 \\ \mu_2 + \alpha_2 Z_{t-1} + \sum_{h=1}^{\ell} \Phi_{2h} \Delta X_{t-h} + \varepsilon_t & \text{if } \gamma_1 < Z_{t-1} \le \gamma_2 \\ \mu_3 + \alpha_3 Z_{t-1} + \sum_{h=1}^{\ell} \Phi_{3h} \Delta X_{t-h} + \varepsilon_t & \text{if } Z_{t-1} > \gamma_2 \end{cases} \quad (1)$$

where t = 1,…,T, α_M refers to a (2 x 1) vector of adjustment coefficients in regime M, μ_M is a regime-specific vector of intercepts, Φ_{Mh} denotes a (2 x 2) regime-specific matrix of the short-run coefficients with lag h, and $\varepsilon_{.t}$ is a (2 x 1) vector of i.i.d. errors. The threshold variable is defined as the error-correction term $Z_{t-1} = \beta' X_{t-1} = P_{1t-1} - P_{2t-1}$ that is equal to the value of the deviations from the long-run equilibrium with the known cointegrating vector β = (1,-1)' implied by the

[6] The three-regime threshold models have been popularly applied in the literature. For instance, Seo (2003) suggested a three-regime TVECM to study the term structure of interest rate in the presence of transaction costs. Chen and Lee (2008) applied a three-regime TAR model to study the price convergence for wholesale hog markets in Taiwan with a nonlinear adjustment process. Wu et al. (2009) adopted a three-regime symmetric band-TAR model to estimate the nonlinear mean reversion of real pound-dollar exchange rate.

PPP theory. The adjustment process toward equilibrium depends on the magnitude of the PPP deviations. The threshold parameters $\gamma = (\gamma_1, \gamma_2)$ satisfying $\gamma_1 \leqq \gamma_2$ take values on a compact set Γ, and the magnitudes of the thresholds represent the proportional transaction costs that delineate different regimes. The thresholds can be asymmetric (i.e., $-\gamma_1 \neq \gamma_2$). This implies that the transaction costs may be lower for goods to arbitrage in one direction than in the opposite direction because, for example, transaction facilities and transportation infrastructure may reduce the cost for a commodity to flow in one direction than in the other (Goodwin and Piggott, 2001). Moreover, all of the slope coefficients can switch between regimes. Therefore, equation (1) includes constrained versions of the three-regime TVECM concerning the price adjustment processes in the presence of transaction costs. Specifically, the transaction-cost theory of goods arbitrage shows that a neutral or threshold band exists in the middle regime ($\gamma_1 < Z_{t-1} \leqq \gamma_2$), inside which deviations from PPP are too small to induce profitable arbitrage. Then, the related prices are not cointegrated and do not tend to move back to equilibrium. When deviations from parity are in the upper or lower regimes, which are defined by $Z_{t-1} \leqq \gamma_1$ and $Z_{t-1} > \gamma_2$ respectively, market forces make the related prices move together and revert toward equilibrium. This implies cointegration with asymmetric adjustment processes in the outer regimes ($\alpha_1 \neq \alpha_3$, $-\mu_1 \neq \mu_3$, and $\Phi_{1h} \neq \Phi_{3h}$). Hence, by imposing the restriction $\alpha_2 = 0$, equation (1) becomes the so-called asymmetric Band-TVECM:

$$\Delta X_t = \begin{cases} \mu_1 + \alpha_1 Z_{t-1} + \sum_{h=1}^{\ell} \Phi_{1h} \Delta X_{t-1} + \varepsilon_t & \text{if } Z_{t\text{-}1} \le \gamma_1 \\ \mu_2 + \sum_{h=1}^{\ell} \Phi_{2h} \Delta X_{t-1} + \varepsilon_t & \text{if } \gamma_1 < Z_{t\text{-}1} \le \gamma_2 \\ \mu_3 + \alpha_3 Z_{t-1} + \sum_{h=1}^{\ell} \Phi_{3h} \Delta X_{t-1} + \varepsilon_t & \text{if } Z_{t\text{-}1} > \gamma_2 \end{cases} \quad (2)$$

It is argued that the thresholds would become symmetric $(-\gamma_1 = \gamma_2)$ when transaction costs are equal if prices are higher in one location or in another; and the price adjustments toward parity in the outer-band regimes would also be symmetric ($\alpha_1 = \alpha_3$, $-\mu_1 = \mu_3$, and $\Phi_{1h} = \Phi_{3h}$) when arbitrage forces operate in the same way if deviations from PPP occur above or below the threshold band (Obstfeld and Taylor, 1997; Juvenal and Taylor, 2008). These restrictions of symmetry, together with the zero restriction on α_2, imply the following symmetric Band-TVECM for X_t:

$$\Delta X_t = \begin{cases} \mu_1 + \alpha_1 Z_{t-1} + \sum_{h=1}^{\ell} \Phi_{1h} \Delta X_{t-h} + \varepsilon_t & \text{if } Z_{t\text{-}1} \le \gamma_1 \\ \mu_2 + \sum_{h=1}^{\ell} \Phi_{2h} \Delta X_{t-1} + \varepsilon_t & \text{if } \gamma_1 < Z_{t\text{-}1} \le \gamma_2 \\ -\mu_1 + \alpha_1 Z_{t-1} + \sum_{h=1}^{\ell} \Phi_{1h} \Delta X_{t-1} + \varepsilon_t & \text{if } Z_{t\text{-}1} > \gamma_2 \end{cases} \quad (3)$$

2.2 Test of threshold cointegration

Balke and Fomby (1997) provided a two-step approach for testing threshold cointegration which consists of testing the null hypothesis of non-cointegration on the univariate cointegrating residuals against the linear cointegration alternative and then testing for the linearity of the cointegrating residuals under the null hypothesis against the alternative of a TAR process once cointegration exists. In this paper, tests of linear cointegration for intranational PPP and threshold nonlinearity in PPP deviations are based on the two-step procedure proposed by Balke and Fomby (1997) but under a multivariate TVECM framework. The multivariate testing methods can utilize the full structure of the model, and so they should have superior power over their univariate counterparts (Lo and Zivot, 2001).

The first step in standard linear cointegration tests is prone to suffer from substantial power decay under the threshold process (Pippenger and Goering, 2000). A new cointegration test is then required to examine the linear non-cointegration null hypothesis in a TVECM, which allows for both the linear and the threshold cointegration alternative. Seo (2006) developed the cointegration test in a TVECM with a known cointegrating vector. If all adjustment coefficients are equal to zero in a TVECM, the linear non-cointegration null cannot be rejected. Seo (2006) considered a special case of Band-TVECM where the intercepts and the higher-order dynamic terms are regime invariant:

$$\Phi(L)\Delta X_t = \alpha_1 Z_{t-1} 1\{Z_{t-1} \le \gamma_1\} + \alpha_3 Z_{t-1} 1\{Z_{t-1} > \gamma_2\} + \mu + \varepsilon_t \qquad (4)$$

where $\mathbf{1}\{.\}$ is an indicator function and $\Phi(L) = I - \Phi_1 L^1 - \ldots - \Phi_\ell L^\ell$. The null hypothesis of $\alpha_1 = \alpha_3 = 0$ in equation (4) is tested using the supremum of the Wald (denoted as Sup-W) statistic:

$$\text{Sup-W} = \sup_{\gamma \in \Gamma} W(\gamma). \qquad (5)$$

Equation (4) is estimated by the least-squares (LS) method. However, the threshold parameters are not identified under the null hypothesis, and this implies that the asymptotic distribution of the Sup-W statistic is nonstandard. Seo (2006) proposed a residual-based bootstrap procedure in order to approximate the distribution of the Sup-W statistic and calculate the associated p-values. We apply the Sup-W statistics for the cointegration tests.

The rejection of the linear no cointegration null hypothesis can be considered as either linear or threshold cointegration, despite the fact that the empirical power of the Sup-W test is larger than that of conventional tests under threshold cointegration. Hence, once the null hypothesis of no cointegration is rejected, we proceed to the second step of testing threshold nonlinearity in the PPP deviations. Lo and Zivot (2001) suggested that Hansen's (1999) method for testing threshold nonlinearity in univariate TAR models can be extended to test the null hypothesis of a linear VECM against the alternative of a three-regime TVECM using the supremum of likelihood ratio (Sup-LR) test statistic:

$$LR_{13} = T(\ln|\hat{\Sigma}| - \ln|\hat{\Sigma}_3|) \tag{6}$$

where $\hat{\Sigma}$ and $\hat{\Sigma}_3$ are respectively the estimated residual covariance matrices of a linear VECM and an unrestricted three-regime TVECM as in equation (1). The problem of unidentified threshold parameters leads to nonstandard distribution of the LR statistics, and then the p-values for the test are obtained using Hansen's (1999) bootstrap procedure. The rejection of the Sup-LR_{13} statistic can be interpreted as evidence of threshold nonlinearity with three regimes (separated by two thresholds) in the adjustment dynamics for X_t.

2.3 Specification testing

Specification testing is important in the threshold analysis of PPP because the transaction cost theory that motivates the empirical versions of the TVECM imposes strong testable restrictions on the model (Lo and Zivot, 2001). We implement specification tests for the TVECM starting with the joint null hypothesis of symmetry, that is, $-\gamma_1 = \gamma_2$, $-\mu_1 = \mu_3$, $\alpha_1 = \alpha_3$, and $\Phi_{1h} = \Phi_{3h}$, h = 1,…,ℓ using the Sup-LR statistic. Under the null, we impose the above symmetric thresholds and coefficients in the outer regimes such that equation (1) is rewritten as:[7]

$$\Delta X_t = \begin{cases} \mu_1 + \alpha_1 Z_{t-1} + \sum_{h=1}^{\ell} \Phi_{1h} \Delta X_{t-1} + \varepsilon_t & \text{if } Z_{t-1} \le \gamma_1 \\ \mu_2 + \alpha_2 Z_{t-1} + \sum_{h=1}^{\ell} \Phi_{2h} \Delta X_{t-1} + \varepsilon_t & \text{if } \gamma_1 < Z_{t-1} \le \gamma_2 \\ -\mu_1 + \alpha_1 Z_{t-1} + \sum_{h=1}^{\ell} \Phi_{1h} \Delta X_{t-1} + \varepsilon_t & \text{if } Z_{t-1} > \gamma_2 \end{cases} \tag{7}$$

The LR statistic denoted as LR_{S3} is computed as

$$LR_{S3} = T(\ln|\hat{\Sigma}_S| - \ln|\hat{\Sigma}_3|) \tag{8}$$

where $\hat{\Sigma}_S$ is the estimated residual covariance matrix of a symmetric TVECM in equation (7). If the joint hypothesis of symmetry is not rejected, we can conclude that

[7] The symmetric three-regime TVECM shown in equation (7) can be directly embedded into the two-regime model by replacing the threshold value Z_{t-1} with its absolute value $|Z_{t-1}|$ (Meyer, 2004 and Seo, 2011).

the thresholds and the adjustments toward PPP parity are symmetric. We then test the null hypothesis of α_2 = 0 in equation (7) using the Wald statistic to examine whether the threshold band exists in the middle regime, where price series exhibit unit root behavior. Seo (2011) showed that the LS estimators of the cointegrating vector β and threshold parameters $\gamma = (\gamma_1, \gamma_2)$ in a TVECM are super-consistent at rate $T^{3/2}$ and T respectively and that the LS estimators of the remaining parameters are asymptotically independent of β and γ, converging to the normal as if the true values of β and γ were known. As a result, the Wald test of zero restriction on α_2 can be computed in the usual way and the Wald statistic follows the asymptotic χ^2 distribution. If the hypothesis of α_2 = 0 is not rejected, it confirms the nonlinear adjustments of the PPP deviations according to the symmetric Band-TVECM as in equation (3). Alternatively, when the joint hypothesis of the symmetry cannot hold but the null hypothesis of α_2 = 0 holds, the asymmetric Band-TVECM as in equation (2) is chosen to model the nonlinear dynamics for X_t.

In addition, according to the Granger representation theorem (Granger, 1986), the cointegration of price variables implies that there is at least one price variable moving to correct the intranational PPP deviations in order to restore the system back to its long-run equilibrium. The error-correction coefficients measure the constant proportions of any deviations from the PPP to be corrected by individual price indices. To examine whether the error-correcting role is taken by either or both of local and benchmark price in the system, we rely upon the statistical significance of the estimated error-correction coefficients (Granger, 1988). The causal impact of the lagged error-correction term that impinges on the long-run relationship of the cointegrated process is considered to be the long-run form of Granger causality (Toda and Phillips, 1993).

2.4 Estimation of mean bias and unconditional half-life

The existence of intranational PPP implies a sufficient degree of mean reversion in the price disparity within a country. The half-life of PPP deviations that is a commonly used measure of PPP persistence would have different policy implications. Half-life is defined as the number of periods it takes for half of the initial effect of a shock to dissipate. It has been suggested that half-life estimates should be computed from the impulse response functions and supplemented with bootstrap confidence intervals which offer a measure of variability for the point estimates (Cheung and Lai, 2000). Impulse response functions are used to measure the effect of a shock occurring at time t on the time series after n horizons. Given that the price adjustments toward the intranational PPP level are nonlinear (either symmetric or asymmetric), half-life estimates and their confidence intervals can be calculated from nonlinear impulse response functions. Unlike a linear model, the nonlinear impulse responses are dependent upon the history or initial condition of the shock, the sign and the size of the shock, the regime in place when the shock hit the market, and the shocks that occur in intermediate periods. As a consequence, there is no unique half-life measure for nonlinear models. Moreover, as shown by the simulation experiment in Lo (2008), with randomized future shocks and the resulting cross-regime dynamics in nonlinear models, it is difficult to predict the impulse responses. It is doubtful whether it is appropriate to use half-life estimates based on regime-specific data as a persistence measure of nonlinear PPP deviations and also whether it can be reasonable to compare the regime-specific or conditional half-life estimates in nonlinear models with the full-sample estimates in linear models. Instead, Lo (2008) proposed a method of computing the mean bias in measuring non-regime specific or unconditional half-life estimates under a nonlinear framework.

The estimates of mean bias are obtained through a simulation of the generalized impulse (GI) response functions developed by Koop et al. (1996). GI response function is defined as the difference between two conditional expectations:

$$GI_X(n, V, \Delta, \Omega) = E(X_{t+n} \mid v_t + \delta_t, v_{t+1} + \delta_{t+1}, ..., v_{t+n} + \delta_{t+n}, \Omega) \\ - E(X_{t+n} \mid v_t, v_{t+1}, ..., v_{t+n}, \Omega) \qquad (9)$$

where history or initial condition $\Omega = (X_{t-1}, X_{t-2}, ..., X_{t-(\ell+1)})$, randomized shocks $V = (v_t, v_{t+1}, ..., v_{t+n})$, and shocks of interest $\Delta = (\delta_t, \delta_{t+1}, ..., \delta_{t+n})$. History Ω and randomized shocks V are irrelevant to linear GI responses, which is denoted as $GI_X(n; \Delta)$. The mean bias (MBI) is defined by Lo (2008) as

$$MBI_X(n,V;\Delta_j,\Omega_j) \equiv GI_X(n;\Delta_j) - \frac{\sum_{k=1}^{K} GI_X(n,V_k,\Delta_j,\Omega_j)}{K} \qquad (10)$$

The estimate of $MBI_X(n,V;\Delta_j,\Omega_j)$ is computed through a Monte Carlo simulation of linear and nonlinear GI response functions based on the same shocks of interest Δ. In the Monte Carlo experiment, Δ_j and Ω_j are the jth set of shocks of interest and history drawn from specific distributions. The second term on the right-hand side of (10) is the mean of the K simulated nonlinear GI response functions with fixed Δ_j and Ω_j but randomized V_k, where subscript k denotes the sub-trial in the nonlinear GI response simulation within each Monte Carlo simulation trial. The first term, $GI_X(n;\Delta_j)$, is the simulated linear GI response function which is shock and history independent. We estimate the mean bias in a TVECM and focus upon the effect of a shock in both the local and benchmark price equations at time t, that is $\Delta = (\delta_t, 0, \ldots, 0)$, on the persistence of nonlinear PPP deviations. All shocks at time t + n, for n > 0, are equal to zero. In each trial, we first randomly draw $\beta' X_{t-1} = P_{1t-1} - P_{2t-1}$ as the initial condition from the empirical distribution. The randomized shocks V_k's are also drawn in a similar manner that may depend on the regime. The unconditional half-life measure of nonlinear PPP deviations after a shock to local or benchmark price in the system can be found by solving for n that satisfies

$$|\hat{GI}_{\beta' X}(n;\Delta) - MBI_{\beta' X.\alpha}(n,V;\Delta,\Omega)| = 0.5 \qquad (11)$$

where $\hat{GI}_{\beta' X}(n;\Delta)$ is the estimated linear GI response functions using the actual data[8] and $MBI_{\beta'X.\alpha}(n, V; \Delta, \Omega)$ denotes the values of the mean bias at the α-*th* percentile of the Monte Carlo simulation for each n. From the definition of the GI response function given in (9), $GI_{\beta'X}(n, V, \Delta, \Omega)$ can be obtained directly as $\beta' GI_X(n, V, \Delta, \Omega)$. Hence, we can calculate $MBI_{\beta'X}(n, V; \Delta, \Omega)$ accordingly from (9) and (10).

2.5 Absorption measures of shocks

Another way to investigate the propagation of shocks in nonlinear models is to measure and analyze the absorption of shocks using the method introduced by van Dijk et al. (2007), which is used to assess how fast shocks are absorbed, that is, the rate at which the final impulse response is attained. With the assumption of V = 0 and $\Delta = (\delta_t, 0, ..., 0)$, the indicator function is defined as

$$I_X(\pi, n, \Delta, \Omega) = \begin{cases} 1 & \text{if } |GI_X(n, \Delta, \Omega) - GI_X^{\infty}(\Delta, \Omega)| \le \pi |GI_X(0, \Delta, \Omega) - GI_X^{\infty}(\Delta, \Omega)| \\ 0 & \text{otherwise} \end{cases} \quad (12)$$

for certain π such that $0 \le \pi \le 1$. The function $I_X(\pi, n, \Delta, \Omega)$ is equal to 1 if the absolute difference between the GI responses at horizon n and the final response given by $GI_X^{\infty}(\Delta, \Omega)$ falls below or equals a fraction π of the absolute difference between the initial impact of the shock (or the GI response at horizon 0) and the final response. The π-life or π-absorption time of X is defined as

$$N_X(\pi, \Delta, \Omega) = \sum_{m=0}^{\infty} \left(1 - \prod_{n=m}^{\infty} I_X(\pi, n, \Delta, \Omega) \right) \quad (13)$$

$N_X(\pi, \Delta, \Omega)$ refers to the minimum horizon beyond which the difference between the impulse responses at all longer horizons and the final response falls below or equals a

[8] Our estimated linear impulse responses for PPP deviations are based on a linear VECM(ℓ), not on a univariate AR(1) model as in Lo (2008).

fraction π of the difference between the initial impact and the final response.

van Dijk et al. (2007) also proposed asymmetric absorption measures and common absorption measures. Let Δ_i^+ and Δ_i^- respectively denote the positive and negative shocks in the equation of P_i, for i = 1, 2. If positive shocks Δ_i^+ and negative shocks Δ_i^- are absorbed at the same rate on average, the asymmetric absorption between the positive and negative shocks of P_i on P_i, for i = 1, 2, in a multivariate model, which are defined respectively as

$$\mathrm{ASYN}_{P_i}(\pi,\Delta_1,\Omega) = N_{P_i}(\pi,\Delta_1^+,\Omega) - N_{P_i}(\pi,\Delta_1^-,\Omega) \quad (14)$$

$$\mathrm{ASYN}_{P_i}(\pi,\Delta_2,\Omega) = N_{P_i}(\pi,\Delta_2^+,\Omega) - N_{P_i}(\pi,\Delta_2^-,\Omega), \quad (15)$$

should have a distribution with a mean equal to 0. Furthermore, when shocks (positive or negative) are absorbed at the same rate by different variables in the system, the common absorption measures, defined as the differences between the π-absorption times of P_1 and of P_2:

$$\mathrm{CN}_{P_1.P_2}(\pi,\Delta_i^+,\Omega) = N_{P_1}(\pi,\Delta_i^+,\Omega) - N_{P_2}(\pi,\Delta_i^+,\Omega) \quad (16)$$

$$\mathrm{CN}_{P_1.P_2}(\pi,\Delta_i^-,\Omega) = N_{P_1}(\pi,\Delta_i^-,\Omega) - N_{P_2}(\pi,\Delta_i^-,\Omega) \quad . \quad (17)$$

should be zero, for i = 1, 2. Likewise, if the shocks are not absorbed differently by the linear combinations $\beta' X$ in the system than by the individual components themselves on average, the alternative common absorption measures that are defined as

$$\mathrm{CAN}_{P_i.\beta'X}(\pi,\Delta_i^+,\Omega) = N_{P_i}(\pi,\Delta_i^+,\Omega) - N_{\beta'X}(\pi,\Delta_i^+,\Omega) \quad (18)$$

$$\mathrm{CAN}_{P_i.\beta'X}(\pi,\Delta_i^-,\Omega) = N_{P_i}(\pi,\Delta_i^-,\Omega) - N_{\beta'X}(\pi,\Delta_i^-,\Omega) \quad (19)$$

should be equal to 0 on average for all i = 1, 2.

3. Data and Empirical Results[9]

3.1 Data

Price index data can provide a wider coverage of all goods traded in the market than a group of individual commodity prices and hence are less likely to be affected by the marketing behavior of one or a few manufacturers or wholesalers which can distort the effect of arbitrage forces on prices. As a price index provides aggregate price information, it contains more relevant implications for monetary and other macroeconomic policies. Using it to test for price convergence makes the results applicable to the problems faced by monetary and macroeconomic policy makers, who generally focus on aggregate inflation rather than on the behavior of individual commodity prices (Cecchetti et al., 2002). Therefore, from the macroeconomic perspective, tests of PPP using price indices are more appropriate than tests of the law of one price using individual product prices.[10]

3.2 Empirical Results

Based on the above, in this paper we empirically examine the convergence of the consumer price indices (CPI) between pairs of cities in Canada. The data series collected from Statistic Canada consist of the monthly all-items CPI for 18

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[9] All empirical results were performed using the GAUSS codes provided by Seo (2006) and were downloaded from Hansen's web page http://www.ssc.wisc.edu/~bhansen, from Lo and Zivot (2001) http://129.3.20.41/md/2001-v5.4/lo-zivot, and from Lo (2008) http://www.degruyter.com/view/j/snde.

[10] Imbs et al. (2005) argued that the aggregation bias leads to upwardly biased estimates of PPP persistence when aggregate data are used. However, Chen and Engle (2005) and Gadea and Mayoral (2009) challenged this argument, concluding that applied macroeconomists can still rely on aggregate data for their studies.

Canadian cities, including Calgary (CAL), Charlottetown and Summerside (CS), Edmonton (EDM), Halifax (HAL), Montreal (MON), Ottawa-Gatineau (OTT), Quebec (QUE), Saint John (SAJ), Saskatoon (SAS), St. John's (STJ), Toronto (TOR), Thunder Bay (TB), Vancouver (VAN), Victoria (VIC), Whitehorse (WHI), Winnipeg (WIN), and Yellowknife (YEL).[11] In this study, all price indices are taken in natural logarithm, demeaned, and seasonally adjusted. Toronto, the largest city in Canada in terms of population, is taken as the benchmark city; and deviations from the intranational PPP are measured by the difference between the local price index (P_{1t}) and the price index of Toronto, the benchmark city (P_{2t}). There are 17 bivariate systems of price indices. The sample periods span the period from December 1984 to May 2010, and the number of observations is 306.

As the first step of the threshold cointegration analysis, we test whether P_{1t} and P_{2t} are cointegrated in each of the 17 bivariate systems with the prespecified cointegrating vector $\beta' = (1,-1)$' using Seo's (2006) Sup-W test. We show the results together with the conventional augmented Dickey-Fuller (ADF), the covariate ADF (CADF), and Horvath-Watson (1995) Wald (H-W Wald) test statistics for comparison.[12] Table 1 reports the results of the cointegration tests. The univariate unit root tests produce similar results to those obtained from the H-W Wald test,

[11] The exclusion of the CPI series for Iqaluit is due to a lack of available data until December 2002.

[12] CADF is proposed by Hansen (1995) for univariate unit root testing with covariates in the ADF regression to increase power. H-W Wald test is a Wald statistic for testing the null of zero adjustment coefficients in a linear VECM with a prespecified cointegrating vector.

which indicate that no more than seven systems of price indices exhibit cointegration. Nevertheless, the results of the Sup-Wald test are the bootstrap p-values which indicate that the null of linear no cointegration is rejected for 13 bivariate systems out of 17, providing evidence of intranational PPP within Canada in most cases. The overwhelming number of rejections produced by the Sup-W test can be found because, as shown in Seo's (2006) simulation study, the power of Sup-W dominates the powers of other tests that ignore the threshold effects under the alternative hypothesis. Furthermore, we undertake a simulation study to show that the power superiority of the Sup-W test can remain no matter whether the prespecified cointegrating vector is correctly imposed in a TVECM or not; the simulation results are reported in Table A.1 of Appendix A. Also, none of the above tests can reject the null of non-cointegration for the four systems of CAL-TOR, SAS-TOR, TB-TOR, and VIC-TOR; thus, those systems are dropped from further analyses.

For the 13 bivariate systems that reject non-cointegration under the null, the empirical study proceeds with multivariate threshold nonlinearity tests and model specification tests as the second and the third steps of the threshold cointegration analysis, respectively. The results are summarized in Table 2. The Sup-LR_{13} test is equivalent to testing the null hypothesis of linearity against equation (1) by imposing the restrictions of $\gamma_1 = \gamma_2 = 0$, $\mu_1 = \mu_2 = \mu_3$, $\alpha_1 = \alpha_2 = \alpha_3$, and $\Phi_{1h} = \Phi_{2h} = \Phi_{3h}$ for $h = 1,\ldots,\ell$. The bootstrap p-values of the Sup-LR_{13} statistics based on equation (6) reject the null hypothesis of linearity in all 13 cointegrated systems, and so we can conclude that the price adjustment dynamics in a TVECM can be characterized by a three-regime, two-threshold process. Once the presence of threshold effects is confirmed, we next examine what kind of threshold model is more appropriate for the price data of each city pair under study. The results of the symmetric tests are based on the p-values of the Sup-LR_{S3} statistics constructed from equation (8), which fail to reject the null hypothesis of symmetry for 8 out of the 13 cointegrated systems (EDM-TOR, MON-TOR, OTT-TOR, QUE-TOR, REG-TOR, SAI-TOR, STJ-TOR, and WIN-TOR), showing evidence of symmetric thresholds and symmetric price convergence toward equilibrium in the outer regimes. In other words, the Sup-LR_{S3} statistics reject the null hypothesis of symmetry for the remaining systems (CS-TOR, HAL-TOR, VAN-TOR, WHI-TOR, and YEL-TOR) in favor of asymmetric thresholds and asymmetric mean reversion for the price indices. Hence, we suggest that the restriction of symmetry in the thresholds and the adjustment process should not be assumed a priori as in previous empirical studies (e.g., Obstfeld and Taylor, 1997; Juvenal and Taylor, 2008; Wu et al. 2009). Finally, the Wald tests of zero α_2 in equation (7) cannot reject the null for all cointegrated systems. The results indicate that the threshold band exists in the middle regime where the goods arbitrage activities cease. Hence, the above results show that there are 8 systems characterized by the symmetric Band-TVECM shown in equation (3) and 5 systems represented by the asymmetric Band-TVECM shown in equation (2).

The symmetric and asymmetric band models are estimated using the sequential conditional LS method (Hansen, 1999). We examine whether the disequilibrium is corrected by the adjustments in the local or benchmark (Toronto) price index based on the significance of the estimated error-correction coefficients in the TVECM using t-ratio statistics, and investigate the long-run form of Granger causal links between the price indices. The results are shown in Table 3. For the cases represented by the symmetric Band-TVECM, we report the error-correction term coefficients in the upper regime (α_1) only because the coefficients in the upper and lower regimes are the same. On the other hand, we report the error-correction term coefficients in both the upper regime (α_1) and the lower regime (α_3) of the asymmetric Band-TVECM. From Table 3, it can be seen that the results regarding the long-run causal links between the local and benchmark price indices in the system are mixed among the city pairs under study. In particular, the coefficients of the error-correction terms are significant in the benchmark price equation only for the MON-TOR, SAJ-TOR, and STJ-TOR systems, and so the long-run form of Granger causality runs from local prices to the benchmark price through the error-correction terms. For OTT-TOR and REG-TOR, only the local price takes the error-correcting role to eliminate PPP deviations, with the long-run Granger causality running from the benchmark price to the local price. For EDM-TOR and WIN-TOR, there is a long-run, bidirectional Granger causal link between the local and benchmark prices as the error-correction coefficients are significant in both price equations, implying that both the local and benchmark price indices contribute to the nonlinear price convergence through the error-correction process. The directions of the long-run feedback effects in the above systems can apply to both outside-band regimes, where the dynamics are symmetric in nature. As for the CS-TOR, HAL-TOR, VAN-TOR, WHI-TOR and YEL-TOR systems, the sources of adjustment for the nonlinear convergence of PPP deviations in each outer regime may not be the same due to the asymmetric dynamics outside of the band. For VAN-TOR and WHI-TOR, error-correcting price adjustment occurs in one regime only but for CS-TOR and YEL-TOR, there is at least one price taking the error-correcting role in both of the upper and lower regimes. From the above, the existence of the dynamic causal relationship between the city price indices through the lagged disequilibrium terms in the TVECM confirms the cointegrated nature of the price indices and provides more evidence of nonlinear mean reversion in the system.

We measure the persistence of shocks using mean bias estimates and nonlinear GI response functions. Following Lo (2008), we first simulate data using the estimated parameters and residuals of the chosen (symmetric or asymmetric) Band-TVECM according to the results in Table 2. In each trial, we use the simulated data to generate linear GI responses $GI_{\beta'X}(n;\Delta_j)$ and mean bias $MBI_{\beta'X}(n,V;\Delta_j,\Omega_j)$ as defined in (9) and (10). On the other hand, we calculate the linear impulse responses $GI_{\beta'X}(n;\Delta)$ using the actual data series in order to calculate the half-life of linear PPP deviations (HL_L). The estimated linear GI responses $GI_{\beta'X}(n;\Delta)$ and the Monte Carlo mean of the mean bias denoted by $\overline{MBI_{\beta'X}}$ are used together to calculate the adjusted nonlinear GI response functions and the unconditional half-life estimates of nonlinear PPP deviations (HL_{NL}) under (11). The corresponding 90%, 95%, and 99% confidence intervals (CI_{NL}) are calculated in a similar way based upon the values of mean bias at the different percentiles, that is, $[MBI_{\beta'X,0.05}, MBI_{\beta'X,0.95}]$, $[MBI_{\beta'X,0.25}, MBI_{\beta'X,0.975}]$ and $[MBI_{\beta'X,0.005}, MBI_{\beta'X,0.995}]$, respectively. We plot the estimates of the adjusted nonlinear GI responses with the 95% confidence intervals in Figures 1 and 2. The adjusted nonlinear GI responses tend to move downward as horizons increase, indicating mean reversion after a shock to the individual prices. Also, the adjusted nonlinear GI responses and the confidence intervals are found to be fluctuating in some cases because of the non-monotonic movements of the mean bias.

The results for the half-lives after the local and benchmark price shocks are reported in Tables 4 and 5 respectively. The empirical confidence intervals (CI_{NL}) can be used to assess precision for the half-life estimates (HL_{NL}) and for hypothesis testing. The unconditional half-life estimates for nonlinear PPP deviations vary across the city pairs under study and in most cases, lying between 1 and 3 years. This represents a lower intranational PPP persistence than that found in studies in the literature on Canada and other countries (e.g., Dayanandan and Ralhan, 2005; Nath and Sarkar, 2009) and indicates a faster speed of mean reversion than the consensus figure of 3 to 5 years for international price differentials (Rogoff, 1996). We also find that the widths of the threshold bands shown in Table 3 tend to be positively correlated with the unconditional half-life estimates for nonlinear PPP deviations being equal to 0.245, indicating that the higher transaction costs implied by the band widths are generally associated with longer half-life estimates. Further, the ranges for the lower and upper bounds of all confidence intervals are mostly less than one year, reflecting the high precision of the half-life estimates.

In addition, we can compare the unconditional half-lives of nonlinear PPP deviations (HL_{NL}) with the linear estimates (HL_L) because they are estimated with the full-sample data. The differences between these two half-lives that are affected by the sizes and signs of the mean bias estimates are found to be statistically significant in many cases. Hence, the mean bias estimates should not be ignored when we calculate the speed of nonlinear mean reversion in PPP deviations. Also, in most cases, the threshold models suggest faster adjustments with shorter half-lives in response to PPP deviations than the linear model. Nevertheless, it is worth noting that there are still the HAL-TOR, OTT-TOR, and STJ-TOR cases, where the half-lives under the threshold models are statistically significantly longer than the linear estimates, implying the slower speed of nonlinear equilibrium adjustments, even though the differences are small and less than half a year. Also, there are cases in which differences between the nonlinear mean estimates and the linear estimates of half-life are statistically insignificant. In such circumstances, the nonlinear adjustment dynamics are well captured and approximated by the linear specification with negligible mean bias estimates. Our above results support Taylor's (2001) claim that inappropriate linear specification may result in biased half-life estimates if the true adjustment process is nonlinear. However, linear half-lives may both over- and under-estimate the nonlinear PPP persistence depending upon the signs of the mean bias estimates (Lo, 2008).[13]

The propagation of shocks can be further assessed by the absorption measures of shocks in both the local price and the benchmark price equations of a Band-TVECM. We set n = 60 (months) as the final horizon to approximate the final impulse responses. Absorption measures may be sensitive to the sizes of shocks and the values of π (van Dijk et al., 2007). We therefore compute the absorption measures for π = 0.7, 0.5, and 0.2, with the sizes of shocks being set to be one-, three-, and five-standard deviation innovations. The history of the series and standard deviation innovations are drawn for the absorption measures in a similar way as for the persistence measures. The hypothesis testing for the mean of the absorption measures relies on the significance of the t-ratio. In line with the argument of Lo (2008), we estimate the full-sample absorption measures, rather than the regime-specific estimates as in van Dijk et al. (2007).

[13] This is consistent with Kilic's (2009) simulation evidence for specific nonlinear models.

To save space, we report the results for π = 0.5 only in Tables 6 and 7, but the analysis below is based on all cases for π = 0.7, 0.5, and 0.2.[14] The results show that the means of the absorption measures vary with the sizes of shocks, values of π, the city pairs under study, and the sources of individual price shocks. In particular, the means of the asymmetric absorptions, $ASYN_{P_i}(\pi, \Delta_1, \Omega)$ and $ASYN_{P_i}(\pi, \Delta_2, \Omega)$, i = 1, 2, from local and benchmark price shocks respectively are mostly (about 86% of all cases) significantly different from zero. As a result, there is strong evidence of asymmetric absorption effects from positive and negative shocks in the system. Moreover, in the majority of all cases (about 92%), the means of common absorptions, $CN_{P_1.P_2}(\pi, \Delta_i^+, \Omega)$ and $CN_{P_1.P_2}(\pi, \Delta_i^-, \Omega)$, for all i = 1, 2 are statistically significantly far from zero. It is concluded that the effects of shocks, positive and negative, are not absorbed at the same rate on average by the different price variables. Finally, the means of the alternative common absorption measures, $CAN_{P_i.\beta'X}(\pi, \Delta_i^+, \Omega)$ and $CAN_{P_i.\beta'X}(\pi, \Delta_i^-, \Omega)$ for i = 1, 2, are also statistically significantly different from zero in almost all cases (92%). In about 72% of the total, the effects of shocks, both positive and negative, for the linear combination $\beta'X$ or the PPP deviations die out faster than for the component price index series in the system.[15] Hence, in most cases, the linear combination can be viewed as a more stable variable when shocks last for a shorter period. To summarize, the above results show strong evidence of asymmetry in the absorption of different types of shocks on different variables in the system and reveal the complicated nonlinear dynamics in the data.

[14] The results for π = 0.7 and 0.2 are not reported but are available on request.

[15] Over half of the cases where the shocks are absorbed faster by the individual price indices than by the PPP deviations can be found in the systems of QUE-TOR and VAN-TOR.

4. Conclusion

In this paper, we adopt multivariate tests of threshold cointegration to study the intranational PPP between 17 pairs of Canadian cities during the period from 1984 to 2010. We find evidence of nonlinear price index convergence in 13 bivariate systems. Also, symmetric and asymmetric adjustment processes can be found in different pairs of cities. Moreover, the directions of Granger causality between the price indices are identified.

Using Monte Carlo simulations, we estimate the unconditional half-lives for nonlinear PPP deviations, which, in most cases, are shorter than the linear estimates. In addition to the persistence of shocks, we measure the absorption of shocks over the full-sample data. This absorption measure can be viewed as complementary to the persistence of shocks; it should not be considered as a substitute because both examine different aspects of the propagations of shocks. Our results regarding the different absorption measures reveal overwhelming evidence of asymmetric absorption rates on different variables caused by different kinds of shocks. In other words, such evidence indicates the existence of complicated nonlinearities in the price index convergence between pairs of Canadian cities.

Table 1 Cointegration tests with the known cointegrating vector $\beta = (1, -1)'$

City	ADF	CADF	H-W Wald	Sup-W (p-values)
CAL	-0.714	-0.133	7.890	0.104
CS	-3.202**	-2.804**	10.366**	0.024**
EDM	-0.530	-0.660	7.103	0.071***
HAL	-3.075**	-2.702***	10.986**	0.008*
MON	-1.233	-1.190	5.574	0.033**
OTT	-2.211	-1.955	14.170*	0.000*
QUE	-2.448	-1.612	6.356	0.019**
REG	-0.888	-0.647	17.285*	0.041**
SAJ	-2.954**	-3.031**	13.945*	0.039**
SAS	-1.130	-0.935	6.859	0.475
STJ	-3.531*	-3.328**	14.995*	0.048**
TB	-1.999	-1.174	3.055	0.406
VAN	-2.137	-1.612	5.188	0.010*
VIC	-1.586	-1.824	2.186	0.513
WHI	-2.069	-1.568	7.467	0.080***
WIN	-1.832	-1.541	10.258**	0.011**
YEL	-1.784	-1.321	6.875	0.053**

Notes:

The ADF regressions include a fitted intercept. The critical values for the ADF test are -2.571, -2.871, and -3.452 at the 10%, 5%, and 1% significance level, respectively.

The stationary covariates in the CADF regressions with an intercept are constructed using Im's (1996) method. The critical values for the CADF test are taken from Hansen (1995).

The critical values for the Horvath-Watson (H-W) Wald test based on one prespecified cointegrating vector $\beta = (1, -1)'$ in a linear VECM are 8.30, 10.18, and 13.73 at the 10%, 5%, and 1% significance level, respectively.

The results of the Sup-Wald test are bootstrap p-values, which are estimated with the inclusion of lagged terms based on the BIC criteria. The number of bootstrap replications is

1,000.
***, **, and * denote significance at 10%, 5%, and 1% level, respectively.

Table 2 Multivariate threshold nonlinearity tests and model specification tests

City	Sup-LR_{13} (p-value)	Sup-$LR_{S.3}$ (p-value)	Wald
CS	0.011**	0.002*	4.519
EDM	0.028**	0.140	4.065
HAL	0.003*	0.015**	0.324
MON	0.088***	0.409	2.047
OTT	0.002*	0.196	4.149
QUE	0.077***	0.586	4.421
REG	0.077***	0.473	4.192
SAJ	0.005*	0.125	1.483
STJ	0.062***	0.112	2.150
VAN	0.011**	0.002*	3.576
WHI	0.008*	0.008*	0.439
WIN	0.088***	0.535	4.486
YEL	0.012**	0.001*	2.653

Notes:
The results of the Sup-LR_{13} and the Sup-LR_{3S} tests are bootstrap p-values, which are estimated with the inclusion of lagged terms based on the BIC criteria. The number of bootstrap replications is 1,000.
The critical values of the Wald tests are 4.605, 5.991, and 9.210 at the 10%, 5%, and 1% significance level, respectively.
***, **, and * denote significance at 10%, 5%, and 1% level, respectively.

Table 3 Estimation of the Band-TVECM

City	ℓ	Adjustment coefficient vectors		$\gamma = (\gamma_1, \gamma_2)$
		U	**L**	
CS	2	-0.230** (0.089) -0.008 (0.067)	-0.016 (0.021) 0.032** (0.016)	-0.0124,-0.0040
EDM	1	-0.038* (0.014) -0.028* (0.007)		-0.0301, 0.0301
HAL	1	-0.108*** (0.059) -0.028 (0.049)	0.062** (0.028) 0.040*** (0.023)	-0.0041, 0.0037
MON	1	0.004 (0.014) 0.031* (0.012)		-0.0116, 0.0116
OTT	0	-0.040** (0.020) -0.006 (0.019)		-0.0071, 0.0071
QUE	0	0.057** (0.023) 0.091* (0.019)		-0.0233, 0.0233
REG	2	-0.034* (0.011) -0.015 (0.010)		-0.0062, 0.0062
SAJ	1	-0.011 (0.012) 0.023** (0.009)		-0.0017, 0.0017
STJ	1	-0.005 (0.008) 0.021* (0.007)		-0.0028, 0.0028
VAN	1	-0.171** (0.078) -0.145*** (-0.087)	0.005 (0.008) 0.006 (0.009)	-0.0207, -0.0198
WHI	1	-0.046 (0.079) -0.054 (0.066)	0.020 (0.014) 0.029** (0.012)	-0.0193,-0.0092
WIN	1	-0.045* (0.013) -0.020*** (0.012)		-0.0050, 0.0050
YEL	1	-0.001 (0.028) 0.053** (0.026)	0.029*** (0.016) 0.031** (0.014)	-0.0062,-0.0045

Notes:

The lags of the Band-TVECM, ℓ, are based on the BIC criterion.

The upper (lower) elements of the adjustment coefficient vectors correspond to the local (benchmark) price equation.

The figures in the parentheses are the standard error of estimates.

***, **, and * denote significance at 10%, 5%, and 1% level, respectively.

Table 4: PPP half-lives (years) and confidence intervals due to shocks in the local price equation

City	HL_L	HL_{NL}	$90\%CI_{NL}$	$95\%CI_{NL}$	$99\%CI_{NL}$
CS	1.489	1.445	[1.338,1.579]	[1.327,1.616]	[1.282,1.765]
EDM	4.738*	2.451	[1.947, 3.360]	[1.580, 3.385]	[1.575, 4.072]
HAL	1.376*	1.254	[1.242,1.246]	[1.241,1.246]	[1.240,1.247]
MON	2.772*	1.548	[1.428,1.807]	[1.425,1.876]	[1.416,1.997]
OTT	2.427**	2.664	[2.524, 2.862]	[2.499,2.896]	[2.427, 2.919]
QUE	4.202*	2.477	[1.997,2.955]	[1.968, 3.006]	[1.844,3.067]
REG	3.150	3.189	[3.167, 3.210]	[3.165,3.211]	[3.163,3.213]
SAJ	1.958	2.079	[1.897, 2.229]	[1.873, 2.310]	[1.798, 2.393]
STJ	2.210*	2.757	[2.612,2.914]	[2.577,2.956]	[2.504,3.091]
VAN	4.617*	1.126	[1.072,1.184]	[1.061,1.220]	[1.058,1.225]
WHI	4.066*	2.135	[1.944,2.481]	[1.944,2.488]	[1.942,2.504]
WIN	2.362	2.410	[1.974,2.777]	[1.966,2.790]	[1.955,2.840]
YEL	2.885*	1.189	[1.063,1.369]	[1.060,1.385]	[1.059,1.396]

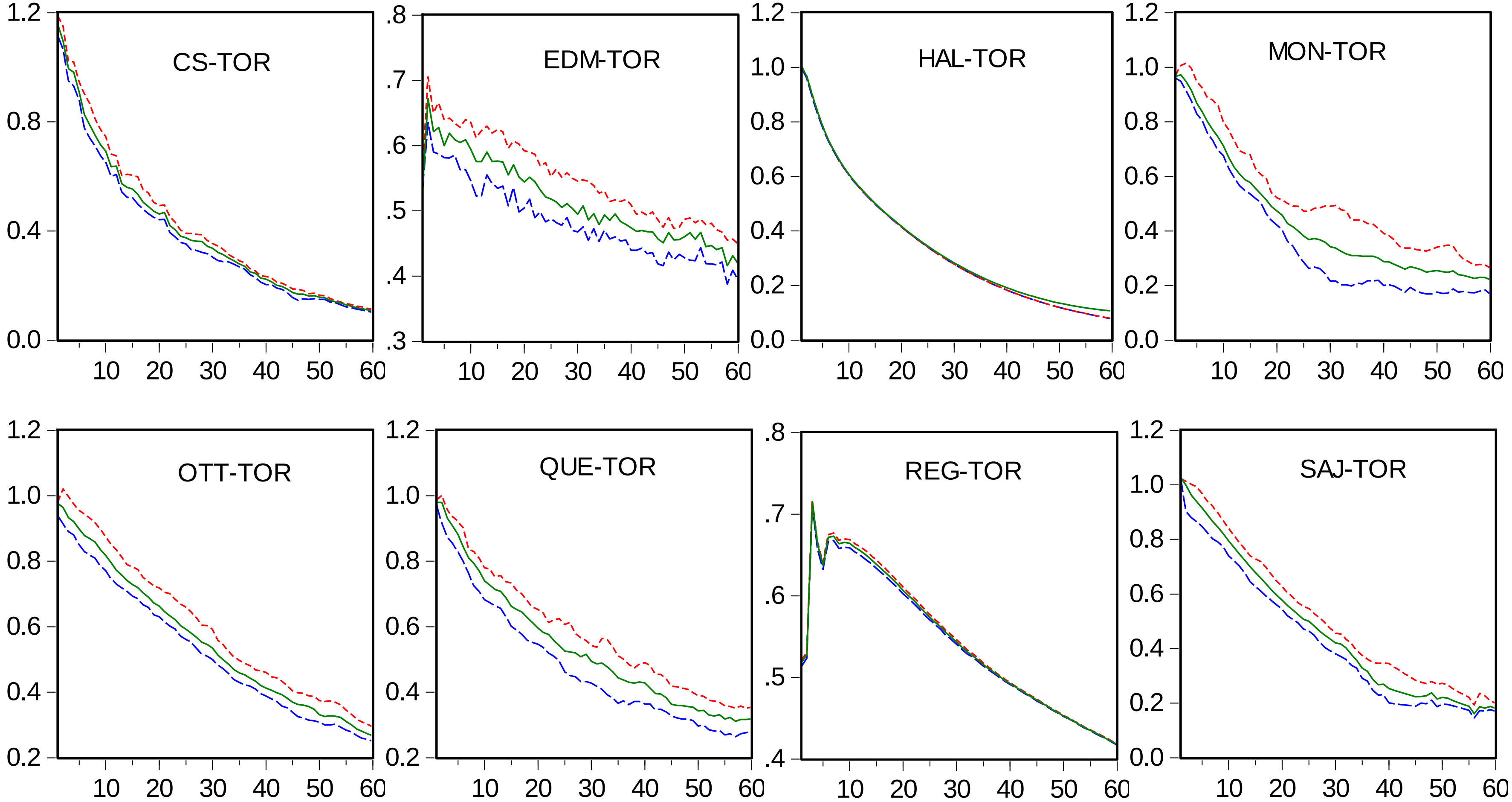
Notes:
$\mathbf{HL_L}$ stands for half-life estimates based on the linear VECM.
$\mathbf{HL_{NL}}$ and $\mathbf{CI_{NL}}$ denote mean estimates and confidence intervals for half-lives of nonlinear PPP deviations, respectively, based on the Band-TVECM.
***, **, and * denote significance at 10%, 5%, and 1% level, respectively.

Table 5: PPP half-lives (years) and confidence intervals due to shocks in the benchmark price equation

City	HL_L	HL_{NL}	$90\%CI_{NL}$	$95\%CI_{NL}$	$99\%CI_{NL}$
CS	0.454*	0.197	[0.160,0.327]	[0.159,0.364]	[0.155,0.393]
EDM	5.260*	1.289	[1.173,1.649]	[1.168,1.693]	[1.159,1.815]
HAL	1.145*	1.329	[1.265,1.367]	[1.263,1.370]	[1.263,1.371]
MON	3.236*	1.696	[1.517,2.198]	[1.503,2.225]	[1.481,2.329]
OTT	2.427*	2.885	[2.690,3.031]	[2.674,3.050]	[2.663,3.067]
QUE	4.202*	2.391	[2.025,2.786]	[2.015,2.796]	[1.869,2.805]
REG	5.827*	3.447	[3.350,3.552]	[3.319,3.583]	[3.293,3.620]
SAJ	1.307	1.457	[1.237,1.605]	[1.226,1.682]	[1.208,1.794]
STJ	2.210*	2.859	[2.659,3.029]	[2.580,3.047]	[2.535,3.076]
VAN	4.804*	1.753	[1.592,1.819]	[1.579,1.825]	[1.503,1.828]
WHI	3.562*	0.416	[0.401,0.448]	[0.401,0.450]	[0.400,0.459]
WIN	2.551	2.817	[2.527,3.334]	[2.502,3.362]	[2.480,3.422]
YEL	2.643*	0.825	[0.755,0.907]	[0.752,0.918]	[0.742,0.940]

See notes to Table 4.

Figure 1. Adjusted nonlinear GI responses with the 95% confidence intervals after local price shocks:



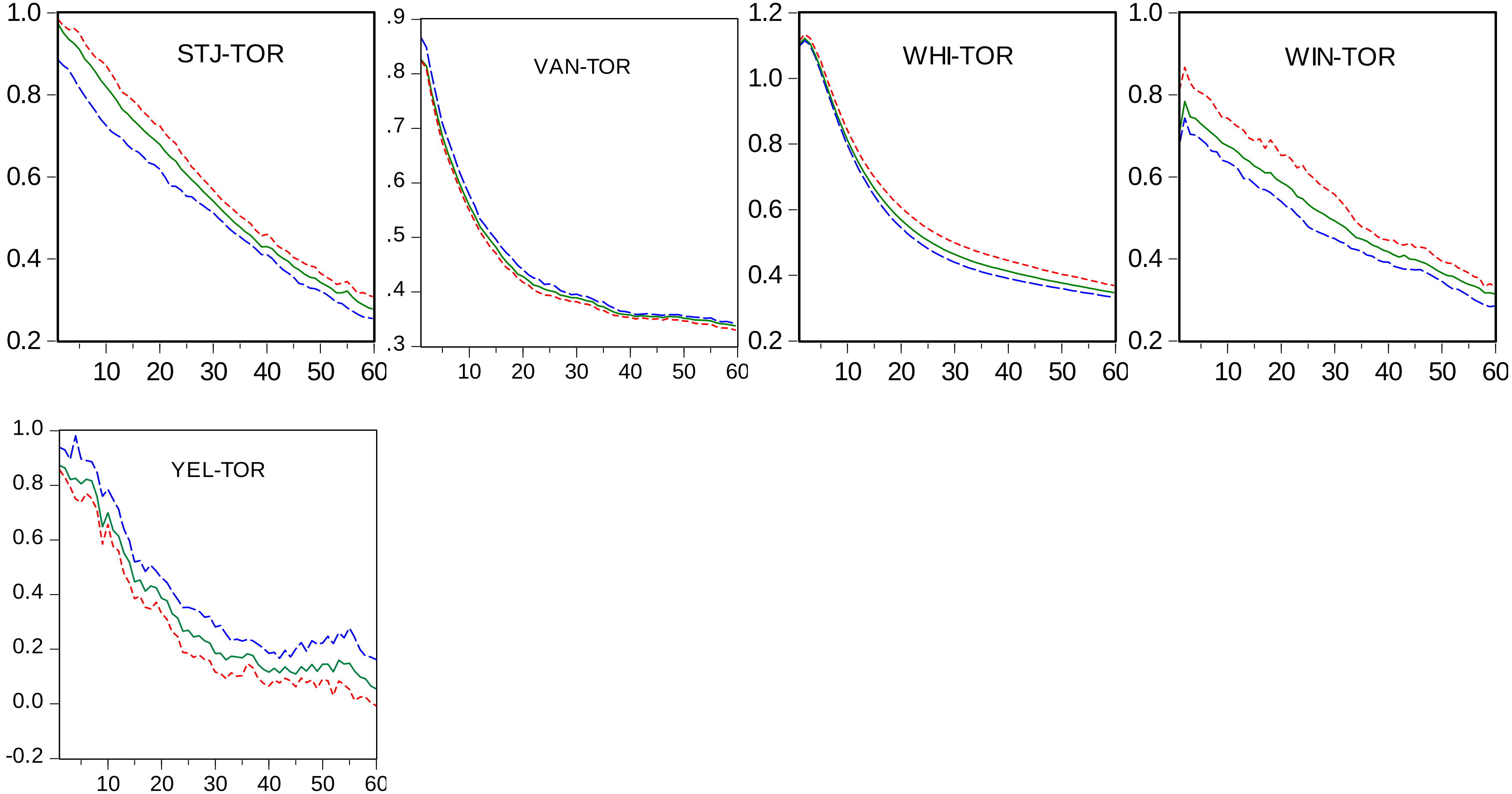
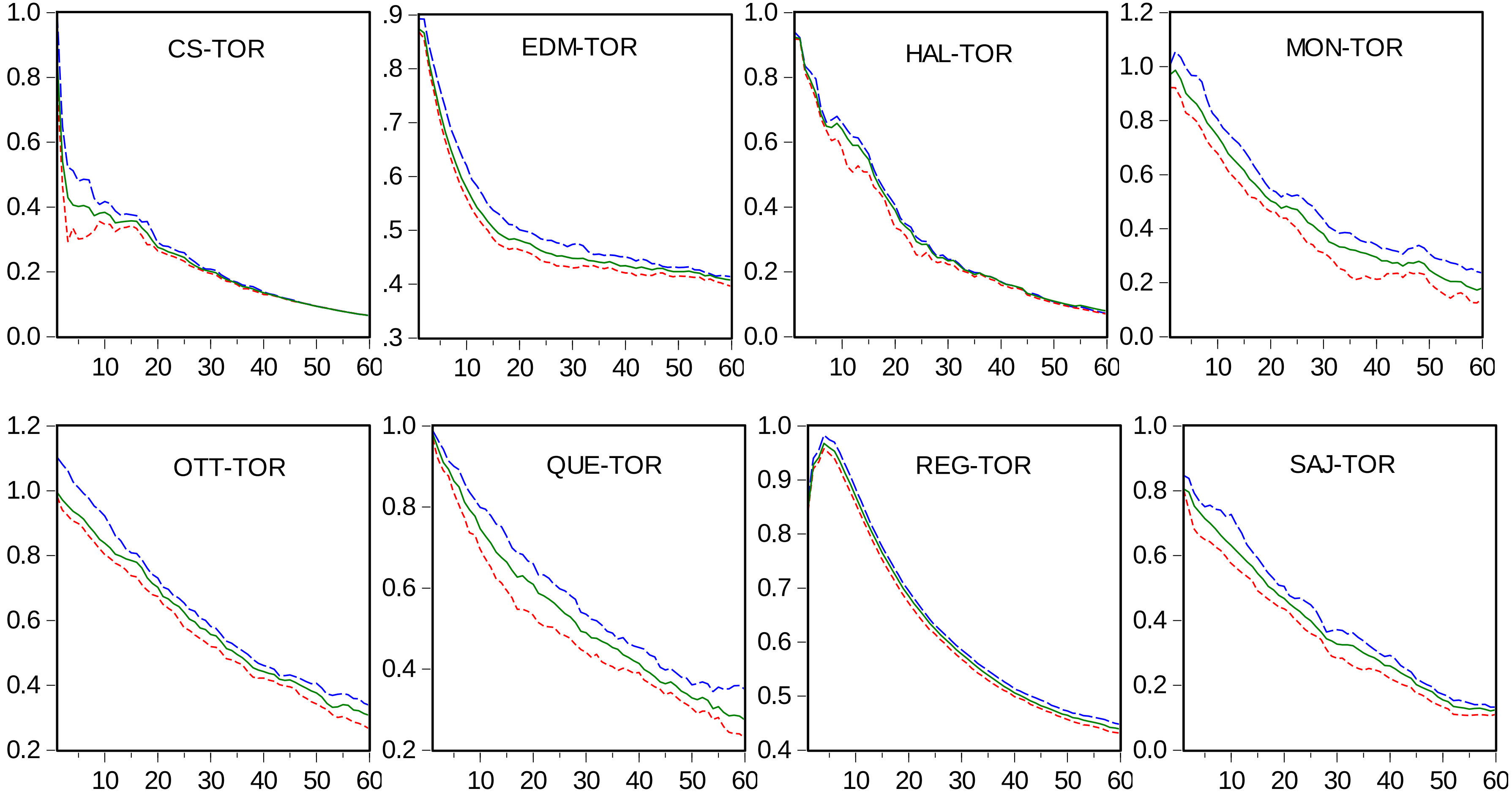

STJ-TOR
VAN-TOR
WHI-TOR
WIN-TOR
YEL-TOR

Figure 2. Adjusted nonlinear impulse responses with the 95% confidence intervals after benchmark price shocks:
:



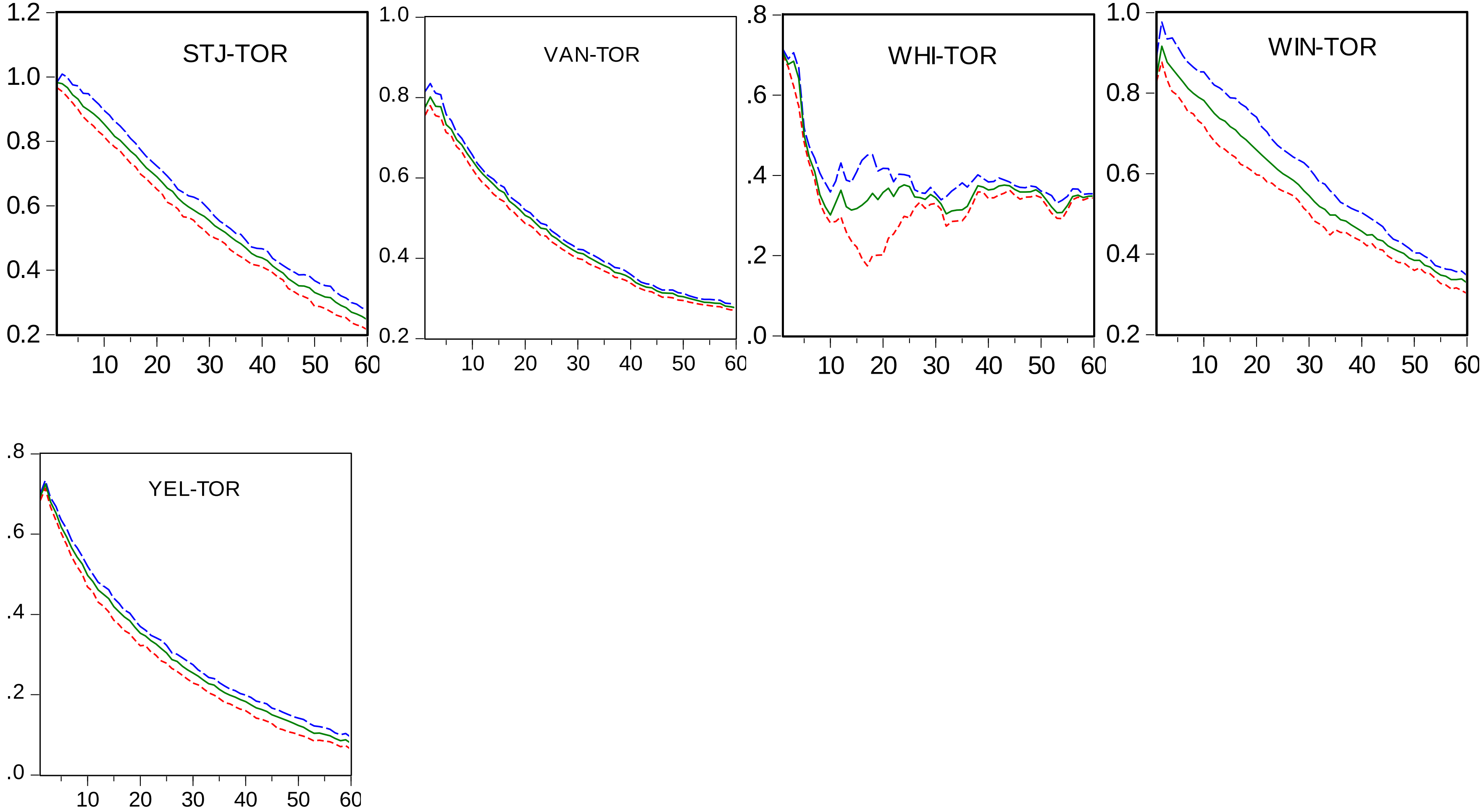

STJ-TOR
1.2
1.0
0.8
0.6
0.4
0.2
10 20 30 40 50 60
VAN-TOR
1.0
0.8
0.6
0.4
0.2
10 20 30 40 50 60
WHI-TOR
.8
.6
.4
.2
.0
10 20 30 40 50 60
WIN-TOR
1.0
0.8
0.6
0.4
0.2
10 20 30 40 50 60
YEL-TOR
.8
.6
.4
.2
.0
10 20 30 40 50 60

Table 6: Asymmetry measures for absorption times (months) for π = 0.5 when a shock takes place in the local price equation

City	Size of Shocks	$ASYN_{P_1}(\pi, \Delta_1, \Omega)$	$ASYN_{P_2}(\pi, \Delta_1, \Omega)$	$CN_{P_1.P_2}(\pi, \Delta_1^+, \Omega)$	$CN_{P_1.P_2}(\pi, \Delta_1^-, \Omega)$	$CAN_{P_i.\beta'X}(\pi, \Delta_1^+, \Omega)$	$CAN_{P_i.\beta'X}(\pi, \Delta_1^-, \Omega)$
CS	1^{σ}	1.221*	0.293**	16.581*	15.653*	21.821*	20.994*
	3^{σ}	7.137*	2.854*	17.262*	12.979*	25.023*	20.199*
	5^{σ}	11.455*	3.002*	17.205*	8.752*	16.603*	13.471*
EDM	1^{σ}	-5.430*	-7.677*	-1.655*	-3.902*	3.061*	3.281*
	3^{σ}	-4.129*	-6.381*	-2.205*	-4.457*	3.049*	8.638*
	5^{σ}	-4.969*	-12.669*	-2.418*	-10.118*	2.500*	13.514*
HAL	1^{σ}	1.072*	-0.438**	1.339*	-0.171	1.479*	0.637*
	3^{σ}	1.301*	0.125	-0.166	-1.342*	-0.374*	0.238**
	5^{σ}	3.433*	-0.554	0.061	-3.926*	-0.357	0.304**
MON	1^{σ}	3.831*	1.159*	5.608*	2.936*	7.450*	3.967*
	3^{σ}	5.627*	5.008*	1.627*	1.008*	3.676*	2.020*
	5^{σ}	5.979*	7.849*	-3.492*	-1.622*	0.976	0.221
OTT	1^{σ}	-4.568*	-2.506*	4.522*	6.584*	1.971*	0.148
	3^{σ}	-14.618*	-15.165*	0.627*	0.080*	2.200*	2.109*
	5^{σ}	-13.006*	-13.111*	1.268*	0.293*	2.933*	2.828*
QUE	1^{σ}	0.726	2.643*	17.745*	19.662*	0.039**	0.008
	3^{σ}	9.384*	12.318*	14.873*	17.807*	-1.089*	-0.568*
	5^{σ}	10.524*	11.249*	10.038*	10.763*	-3.474*	-0.553*
REG	1^{σ}	0.327	4.285*	0.836	-11.605*	-7.647*	4.509*

	3σ	-1.638*	3.075*	-10.582*	-5.869*	4.249*	5.767*
	5σ	-5.703*	5.881*	-15.426*	-3.842*	0.394	6.179*
SAJ	1σ	-1.941*	-1.295*	9.134*	9.780*	9.905*	10.732*
	3σ	-4.770*	-0.909*	5.544*	9.405*	4.024*	11.050*
	5σ	-1.035	-2.511*	9.818*	8.342*	5.973*	13.213*
STJ	1σ	0.253	0.465**	2.027*	2.239*	4.884*	4.469*
	3σ	-3.443*	-4.033*	2.131*	1.541*	5.019*	4.720*
	5σ	0.385	-0.698	1.873*	0.790*	-5.025*	-1.691*
VAN	1σ	3.436*	4.879*	-0.869*	0.574**	2.324*	-4.111*
	3σ	6.224*	6.449*	-0.345	-0.120	0.527*	-0.558*
	5σ	9.112*	11.389*	-0.961*	1.316*	1.432*	-0.807*
WHI	1σ	-0.243	-0.140	-1.749*	-1.646*	2.400*	2.908*
	3σ	3.652*	0.910	-0.485	-3.227*	4.635*	1.052*
	5σ	4.946*	1.522*	0.799**	-2.625*	5.720*	0.491*
WIN	1σ	2.108*	5.177*	-5.571*	-2.502*	11.406*	9.934*
	3σ	2.041*	7.142*	-7.359*	-2.258*	5.171*	3.428*
	5σ	-2.330*	9.485*	-13.468*	-1.653*	-6.045*	1.353*
YEL	1σ	-0.648**	0.766**	-2.275*	-0.861*	1.079*	1.852*
	3σ	0.755**	1.600*	-0.730**	0.115	4.373*	4.227*
	5σ	2.686*	3.467*	-0.872**	-0.091	7.154*	5.322*

Notes:

***, **, and * denote significance at 10%, 5%, and 1% level, respectively.

Table 7: Asymmetry measures for absorption times (in months) for π = 0.5 when a shock takes place in the benchmark price equation

City	Size of Shocks	$ASYN_{P_2}(\pi, \Delta_2, \Omega)$	$ASYN_{P_1}(\pi, \Delta_2, \Omega)$	$CN_{P_1.P_2}(\pi, \Delta_2^+, \Omega)$	$CN_{P_1.P_2}(\pi, \Delta_2^-, \Omega)$	$CAN_{P_2.\beta'X}(\pi, \Delta_2^+, \Omega)$	$CAN_{P_2.\beta'X}(\pi, \Delta_2^-, \Omega)$
CS	1σ	-0.863*	-0.226	-20.732*	-21.369*	17.200*	17.930*
	3σ	-2.647*	-0.770***	-19.137*	-21.014*	16.711*	18.889*
	5σ	-6.034*	-2.454*	-17.384*	-20.964*	14.368*	17.124*
EDM	1σ	2.928*	2.946*	-0.633*	-0.651*	3.554*	3.322*
	3σ	7.549*	7.839*	-0.759*	-1.049*	4.358*	5.280*
	5σ	7.031*	8.894*	-2.644*	-4.507*	6.517*	8.452*
HAL	1σ	-0.381***	-0.521*	0.083	0.223	0.256	-0.154
	3σ	-2.342*	-2.327*	-1.220	-1.235*	0.705*	0.252
	5σ	-3.068*	-4.268*	-1.071*	0.129	0.336***	-1.675*
MON	1σ	0.382	-2.063*	12.396*	14.841*	4.109*	2.008*
	3σ	-4.319*	-1.054***	2.007*	-1.258*	0.798*	1.043*
	5σ	-7.751*	1.613*	4.119*	-5.245*	2.125*	2.832*
OTT	1σ	6.394*	3.979*	-4.332*	-1.917*	0.903*	1.596*
	3σ	14.289*	13.272*	-0.624*	0.393*	1.764*	1.783*
	5σ	13.274*	12.259*	0.405*	1.420*	3.885*	1.963*
QUE	1σ	-0.723	0.807	-12.331*	-13.861*	-0.087***	-0.008
	3σ	-9.271*	-9.842*	-11.415*	-10.844*	-0.267*	-0.452*
	5σ	-9.485*	-8.696*	-9.044*	-9.833*	-0.452*	-2.062*
REG	1σ	-5.644*	-2.613*	1.275	-1.756	3.918*	4.686*

	3^{σ}	-4.999*	2.314*	1.283**	-6.030*	2.586*	-2.553*
	5^{σ}	-6.561*	7.620*	0.378	-13.803*	2.877*	-7.761*
SAJ	1^{σ}	0.478**	-1.165*	-23.178*	-21.535*	14.442*	13.262*
	3^{σ}	5.282*	-4.223*	-20.519*	-11.014*	12.776*	5.324*
	5^{σ}	0.829***	-6.097*	-28.353*	-21.427*	19.635*	14.817*
STJ	1^{σ}	0.163	-1.404**	3.784*	5.351*	2.938*	3.068*
	3^{σ}	3.232*	3.141*	1.799*	1.890*	3.473*	3.142*
	5^{σ}	2.810*	2.387*	1.288*	1.711*	2.542*	2.202*
VAN	1^{σ}	-1.930*	-1.619*	1.384*	1.073*	-0.268	-1.610*
	3^{σ}	-5.561*	-5.418*	1.385*	1.242*	0.969*	2.939*
	5^{σ}	-5.048*	-1.205*	6.170*	2.327*	-8.061*	-3.257*
WHI	1^{σ}	-0.547*	-1.558*	-8.076*	-7.065*	3.496*	1.267*
	3^{σ}	-1.085*	-2.335*	-7.677*	-6.427*	5.443*	0.174
	5^{σ}	-2.161*	-3.208*	-6.155*	-5.108*	7.452*	-1.115*
WIN	1^{σ}	-5.504*	-2.454*	-2.285*	-5.335*	12.369*	14.487*
	3^{σ}	-9.242*	0.492	2.691*	-7.043*	4.761*	9.150*
	5^{σ}	-10.922*	6.953*	1.715*	-16.160*	-3.576*	7.126*
YEL	1^{σ}	-1.453*	1.330*	12.269*	9.486*	0.963*	0.766**
	3^{σ}	-2.962*	1.100***	11.083*	7.021*	4.192*	1.226*
	5^{σ}	-4.759*	0.782	10.7793*	5.252*	5.251*	2.197*

Notes:

***, **, and * denote significance at 10%, 5%, and 1% level, respectively.

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Appendix A: Power comparison of cointegration tests constructed from incorrect values of cointegration vectors

In Engle and Granger (1987) terminology, the necessary condition for PPP (or weak form PPP) is the existence of stationary PPP deviations which are defined as $\beta' X_t = P_{1.t} - bP_{2.t}$ with the cointegrating vector β = (1,-b)’, whereas the necessary and sufficient condition for PPP (or strong form PPP) is the existence of stationary PPP deviations, and the cointegrating vector β is strictly prespecified as (1,-1)’ (Dutt and Ghosh, 1995). The strong form PPP hypothesis may not be supported because the existence of non-tradable goods, shifts in real factors such as differentials in productivity growth and changes in consumption patterns between regions (Sarno, 2008), and also measurement errors (Taylor, 1988) may systematically deviate from the long-run one-for-one relationship of price differentials. From the econometric point of view, it is important to examine the effects on the power of these test statistics when we strictly prespecify the cointegrating vector β = (1,-1)’ in a TVECM which may however be incorrect.[16] Thus, we undertake a simulation experiment to examine the power performance of the Sup-W test statistic, together with the conventional ADF, CADF, and H-W Wald test statistics for comparison, when the prior knowledge of the cointegrating vector may be inexact.

[16] Likewise, Horvath and Watson (1995) examined the effect of the power of the H-W Wald test when the values of the cointegrating vector in a linear VECM are incorrectly prespecified.

To do this, the data $X_t = (P_{1t}, P_{2t})'$ are generated from the following bivariate Band-TVECM with symmetric thresholds:

$$\Delta X_t = (-a,0)' Z_{t-1} 1\{Z_{t-1} \le -\lambda\} + (0,a)' Z_{t-1} 1\{Z_{t-1} \ge \lambda\} + \varepsilon_t, \quad \text{(A1)}$$

where **1**(.) is an indicator function. The different types of threshold effects are constructed by varying λ among {3, 4, 6} and choosing the adjustment coefficient a from among {0.15, 0.25}. Equation (A1) is similar to the simple band-TVECM in the simulation experiment conducted by Seo (2006). However, we set the threshold value $Z_{t-1} = (P_{1.t-1} - bP_{2.t-1})$ with the true cointegrating parameters b varying among {0.7, 0.75, 0.80, 0.85, 0.90, 0.95, 1.00}, but the cointegrating vectors (1,-b)' are all prespecified to be (1,-1)' in the simulation experiments. The error term ε_t follows i.i.d $N(0, \Sigma)$, where $\Sigma = \begin{pmatrix} 1 & 0.8 \\ 0.8 & 1 \end{pmatrix}$. The sample size is 300, which approaches the number of usable observations in our empirical study. The calculation of the Sup-W statistic is based on the two-regime TVECM ($\gamma_1 = \gamma_2$ in equation 4) when the asymptotic distribution of Sup-W does not depend on whether it is computed from two-regime or Band TVECM (Seo, 2006). The number of simulations is 1,000 and the number of bootstrap replications for Sup-W is 500. The empirical power results are reported in Table A.1. If the cointegration vector is correctly specified as (1,-1)' when b = 1, the empirical powers of the cointegration tests under study are the highest in all cases, but decrease gradually with the prespecified values of the cointegrating parameters departing from their true values. Nevertheless, the power of Sup-W is still dominant over the powers of other tests in our simulation study. In testing for PPP in a TVECM where the adjustment process is nonlinear, we still suggest Sup-W for the cointegration test even though the true cointegrating parameters may not be correctly imposed.

Table A.1: Power of cointegration tests when the true cointegration vector (1,-b)' in a bivariate Band-TVECM is prespecified to be (1,-1)'

	Sup-W		**ADF**		**CADF**		**H-W Wald**	
Nominal size	10%	5%	10%	5%	10%	5%	10%	5%
b	a = 0.15; $\lambda = 3$							
1.00	0.985	0.945	0.954	0.763	0.945	0.778	0.860	0.763
0.95	0.955	0.853	0.891	0.676	0.889	0.695	0.784	0.676
0.90	0.824	0.678	0.769	0.550	0.787	0.570	0.680	0.550
0.85	0.662	0.494	0.623	0.434	0.658	0.459	0.591	0.433
0.80	0.542	0.366	0.489	0.310	0.511	0.338	0.484	0.310
0.75	0.439	0.291	0.375	0.234	0.385	0.260	0.396	0.234
0.70	0.361	0.226	0.288	0.181	0.305	0.202	0.335	0.181
b	a = 0.15; $\lambda = 4$							
1.00	0.911	0.827	0.530	0.280	0.542	0.339	0.439	0.280
0.95	0.840	0.711	0.493	0.266	0.520	0.266	0.433	0.266
0.90	0.712	0.580	0.452	0.254	0.485	0.285	0.404	0.254
0.85	0.591	0.458	0.398	0.231	0.417	0.253	0.377	0.231
0.80	0.494	0.335	0.344	0.202	0.356	0.210	0.356	0.202
0.75	0.395	0.257	0.285	0.164	0.296	0.178	0.312	0.164
0.70	0.339	0.228	0.239	0.122	0.254	0.141	0.276	0.122
b	a = 0.15; $\lambda = 6$							
1.00	0.684	0.601	0.267	0.180	0.273	0.182	0.272	0.180
0.95	0.644	0.540	0.259	0.180	0.271	0.177	0.262	0.180
0.90	0.571	0.454	0.255	0.172	0.277	0.172	0.263	0.172
0.85	0.479	0.340	0.242	0.148	0.253	0.160	0.261	0.148
0.80	0.413	0.300	0.236	0.132	0.231	0.140	0.247	0.132
0.75	0.344	0.228	0.211	0.117	0.213	0.124	0.229	0.117

0.70	0.305	0.194	0.186	0.110	0.179	0.111	0.210	0.110
b	a = 0.25; λ = 3							
1.00	0.999	0.997	0.998	0.972	0.998	0.960	0.984	0.972
0.95	0.997	0.976	0.982	0.888	0.977	0.873	0.928	0.887
0.90	0.937	0.858	0.887	0.735	0.891	0.743	0.825	0.735
0.85	0.800	0.677	0.742	0.563	0.757	0.581	0.691	0.562
0.80	0.678	0.538	0.601	0.437	0.607	0.433	0.581	0.437
0.75	0.550	0.390	0.444	0.293	0.455	0.308	0.454	0.293
0.70	0.441	0.308	0.331	0.224	0.340	0.220	0.389	0.224
B	a = 0.25; λ = 4							
1.00	0.965	0.926	0.749	0.424	0.692	0.438	0.603	0.424
0.95	0.929	0.858	0.666	0.384	0.642	0.400	0.543	0.384
0.90	0.829	0.720	0.583	0.354	0.562	0.347	0.512	0.354
0.85	0.727	0.584	0.502	0.302	0.487	0.293	0.466	0.302
0.80	0.597	0.454	0.420	0.252	0.398	0.248	0.425	0.251
0.75	0.487	0.337	0.342	0.198	0.330	0.190	0.357	0.198
0.70	0.407	0.269	0.287	0.155	0.263	0.149	0.312	0.155
b	a = 0.25; λ = 6							
1.00	0.715	0.642	0.295	0.199	0.231	0.138	0.297	0.199
0.95	0.700	0.627	0.288	0.197	0.217	0.138	0.287	0.197
0.90	0.647	0.550	0.274	0.191	0.218	0.138	0.289	0.191
0.85	0.579	0.464	0.264	0.176	0.201	0.119	0.270	0.176
0.80	0.507	0.374	0.259	0.152	0.195	0.118	0.270	0.152
0.75	0.420	0.295	0.230	0.143	0.173	0.102	0.248	0.143
0.70	0.363	0.263	0.202	0.120	0.161	0.095	0.228	0.120

Notes:
The simulation is based on 1,000 replications for a sample size of 300.
The number of bootstrap replications for the Sup-W test is 500.
The critical values for the ADF, CADF, and H-W Wald tests are shown in the notes to Table 1.

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